

## An Introduction to Game Theory

**Game theory** is the study of strategic decision making. More formally, it is "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers who are seeking their self interest". An alternative term suggested "as a more descriptive name for the discipline" is **interactive decision theory**.

We will use Game Theory in this class to supplement the material found in the district curriculum. Although Game Theory is an extension of the district curriculum, it can be aptly applied to economics, politics, and social situations. This makes game theory one of the "bridges" between the different concepts found in the course.

We will study game theory conceptually and not so much mathematically. The goal for unit 1 is to learn the basic models and stories. After unit 1 we will apply the basic models to more concrete ideas of the course. Although we will learn additional games after unit 1, the foundation of our most important games will be established.

Historically, the most advanced students in this course have not only applied game theory to the topics found in all units but also to their core topics and their final exam essay.

Make no mistake about it game theory is a real subject, studied at every university, utilized by the CIA, businesses leaders, economic advisors, and political parties. Scholars have earned the Nobel Laureate in economics (the highest award in the profession) for their use of game theory.

### The Basics

#### Players, Strategies, and Pay-offs

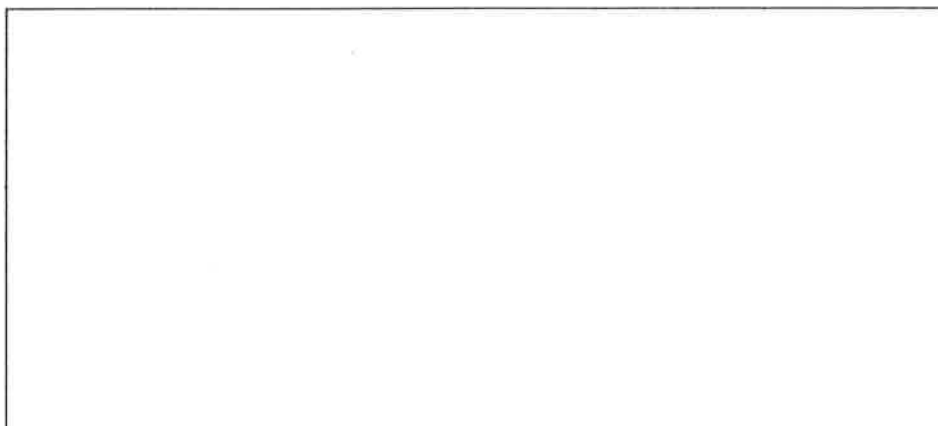
For most games we will develop matrices to help us understand the game.

We will learn the basics of game theory by playing one of the most children's games, Rock-Scissors-Paper.

First it is a 2 player game. Player 1 and Player 2.

**Player 2**

**Player 1**



Next each player has 3 strategies. They can play a Rock, Scissors, or a Paper.

		Player 2		
		Rock	Scissors	Paper
Player 1	Rock			
	Scissors			
	Paper			

We need to make a couple assumption.

1<sup>st</sup>- Both players are trying to win and therefore will make rational decisions to achieve this priority

2<sup>nd</sup>- Both players know the rules

3<sup>rd</sup> – Neither player knows what the other is going to do.

4<sup>th</sup>- Both players know the pay-offs and are trying to maximize their utility

#### **Pay-offs**

In Game theory, payoffs may be explained in specific detail, by assigning a numbers, money, years, or even happy and sad faces. The important thing is that each player knows the impact of their decisions.

**In, the game Rock-Scissors-Paper, both players understand the following:**

**Rock beats Scissors (+1 for Rock, - 1 for Scissors)**

**Scissors beats Paper (+1 for Scissors, -1 for Paper)**

**Paper beats Rock (+1 for Paper, -1 for Rock)**

**All ties result in zero for both players**

By filling in the matrix with the pay-offs it can give you a visual of the basics of this game.

		Player 2		
		Rock	Scissors	Paper
Player 1	Rock	0, 0	+1, -1	-1, +1
	Scissors	-1, +1	0, 0	+1, -1
	Paper	+1, -1	-1, +1	0, 0

As a class we will play Rock-Scissors-Paper. Your goal is to earn as much money as possible.

When playing games students should always think about the following items:

1. Play dominate strategies that best maximize your payoffs. Is there a dominate strategy?
2. Make rational choice based on your information. What information do you know about the game? The other player? The pay-off?
3. Know your self interest. What do you want?
4. Put yourself in other's shoes. What do they want?
5. Eliminate weaker options so that you are left with the Best Response. Is there a dominated strategy?
6. Think strategically- anticipate (look forward and reason backward)
7. Cooperation must be enforced. You're competing

After we play the game please answer the questions below.

1. Was there a specific strategy that always won (no matter what the other player did) or in other words was there a strictly dominate strategy?
2. Was there a specific strategy that always lost (no matter what the other player did) or in other words was there a strictly dominated strategy?
3. What was the best approach to take with this game?
4. For each box what was the sum of each payoff?
5. In life have you ever heard anything referred to as a zero-sum game?

We will now play a different game. This game is even simpler and relates to this class even more than Rock-Scissors-Paper.

The game is called Rock-Scissors and below is the completed matrix.

		Player 2	
		Rock	Scissors
Player 1	Rock	0, 0	+1, -1
	Scissors	-1, +1	0, 0

Now turn to person next to you and play the game three times. After you play the game answer the questions below.

1. Was there a specific strategy that always resulted in a better payoff (no matter what the other player did) or in other words was there a strictly dominate strategy?
2. Was there a specific strategy that always resulted in a worse payoff (no matter what the other player did) or in other words was there a strictly dominated strategy?
3. What was the best approach to take with this game?
4. If you know that your opponent will play rock what should you play?

5. If you know that your opponent will play scissors what should you play?
6. Do you think your opponent is thinking the same thing?
7. Acting as player 1 circle the payoff for playing the correct strategy if player 2 plays rock, repeat if player 2 plays Scissors.
8. Acting as player 2 circle the payoff for playing the correct strategy if player 1 plays rock, repeat if player 1 plays Scissors.
9. Is there a box where both payoffs are circled? If so that is a Nash Equilibrium.

		<b>Player 2</b>	
		<b>Rock</b>	<b>Scissors</b>
<b>Player 1</b>	<b>Rock</b>	<b>0, 0</b>	<b>+1, -1</b>
	<b>Scissors</b>	<b>-1, +1</b>	<b>0, 0</b>

### **Nash Equilibrium**

A concept of game theory where the optimal outcome of a game is one where no player has an incentive to deviate from his or her chosen strategy after considering an opponent's choice. Overall, an individual can receive no incremental benefit from changing actions, assuming other players remain constant in their strategies. A game may have multiple Nash equilibria or none at all.

Put in another way, if you knew that your opponent was playing Rock, you would still play Rock and vice versa. If you knew your opponent was playing scissors you would still play Rock and vice versa.

It is in both player's self-interest to play Rock. But what if for some reason it was in the best interest of society for everyone to play scissors?

What if it was in society's best interest for each player to agree to play their dominated strategy?

This is one of the big questions of the course and fundamental part of government.

		Player 2	
		Pollute	Restrict
Player 1	Pollute	0, 0	+1, -1
	Restrict	-1, +1	0, 0

Consider this scenario. . .

Two drivers, driving on streets that are perpendicular to each other, approach the same intersection at the same time.

This intersection has no stop signs, no traffic lights, and no traffic cop to wave cars through and to tell others to stop.

Consider both drivers have an incentive to get to their destination on time and don't want to be bothered to stop but at the same time neither wants to crash?

**Predict what will happen assuming they have perfect information (they see each other) and that they are acting rational.**

**Do we really need lights? Signs? Or laws?**

**How do we change incentive?**

**Brainstorm other situations that people interact in predictable manners in the absence of law.**

**Why do we need laws?**

**How about this scenario. . .**

<https://www.youtube.com/watch?v=rdIL65LD614>







## Coordinating Individualism to create Society

**Historic Background:** Enlightened Philosopher Jean-Jacques Rousseau believe that by forming society man was improving himself and the others around him. Central to Rousseau's beliefs was the idea of the **general will**. To Rousseau, the general will represented the collective values and wisdom of society. Society is coordinated and unified by the general will.

Rousseau believed that since the general will represented the values of the community it must be administered by a benevolent authority (government). Rousseau also believed that the general will demanded the unqualified obedience of every individual.

Rousseau thought private property to be the source of social ills. He considered that private ownership of property tended to corrupt men and destroy their character and regarded the man without property to be the freest. Although he did not actually support the abolition of private property, he believed that private property should be minimal and should be distributed equally among the members of the society.

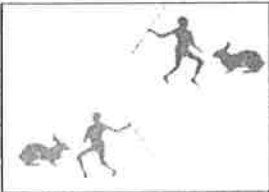



Rousseau appealed to people to surrender their individual rights to a new moral and collective body with one will. Therefore the general will was formed by a social contract. What it willed was the true interest of what everyone wanted whether they realize it or not. According to Rousseau, when you are forced to obey it, you really are obeying yourself, the true and free you.

### Stag hunt

To illustrate his views, Rousseau told the story of the Stag hunt.

Story: Villagers go on a hunt. If they cooperate and hunt the stag (deer) the potential reward is greater than if they hunt on their own and only capture a rabbit. They need to be patient and work together for the stag or they can go out on their own and kill the rabbit which would be easier but would also jeopardize the chances for the other hunters.

The diagram below is the matrix of the story.

		Hunter 2	
		Rabbit	Stag
Hunter 1	Rabbit		
	Stag		

1. What does this story tell us about social cooperation?
2. Why might some people refer to this game as the trust game?
3. Although hunting the Stag may result in the greatest reward, why does it also have the highest risk?
4. Should a hunter be forced to share in the risks? Why or why not?
5. If you believed that the other hunter was going to hunt the Stag what should you do? Why?
6. If you believed that the other hunter was going to hunt the rabbit what should you do? Why?
7. If nobody in the village trusted each other what is the equilibrium point?
8. If there was trust in the village what is the equilibrium point?
9. What would be some methods to better ensure that each hunter would hunt the Stag? Consider methods using the power of the government and methods that don't use government power.
10. To what extent is hunter's sense of the impartial spectator motivation enough to cooperate?
11. Do you feel that the village has the right to tell a specific hunter that they cannot hunt for rabbit if they feel that it will prevent the village from eating a Stag? Explain your logic.
12. Assume the hunters individual cost is the same regardless of what he hunts. What if the hunter's shared portion of the Stag meat (shared portion of the marginal social benefit) amounts to less meat than if he had hunted a rabbit on his own (marginal private benefit). Should he be forced to hunt the stag? Explain your logic.
 

Hunting the rabbit	Marginal Cost= Marginal Benefit
Hunting the Stag	Marginal Private Cost > Portion of the share social Marginal Benefit
13. Relate this story to the individual rights, the original dilemma, reputation, law, and contracts.

14. What do you think Rousseau would consider the "general will" of the story? How would he answer question 10? Explain your logic

15. What if any are the potential negatives of forced cooperation?

16. Using the basic model of "Stag Hunts" found below, brainstorm real situations (current and/or in history) that can relate to this story.

		Player 2	
		Defect	Cooperate
Player 1	Defect	OK, OK	OK, Bad
	Cooperate	Bad, OK	Best for all



## The Ultimatum game

### The Experiment:

In this experiment each of you will be paired with another person in this room. One of you will be designated the proposer, and the other will be designated the responder. You will never be told whom you are paired with; and I will never know who the pairs are. This experimental format is called a "double-blind," and it is designed to make you completely anonymous. You are the only person who will know what decisions you make. In double-blind experiments, participants need not worry about other people judging their actions, so they feel free to act as they truly desire to act.

For the first part of the experiment, which will take only about 3 minutes, the proposers will remain in the room, and the responders will wait silently in the hall. Then, you will silently switch places and the proposers will wait in the hall for about 3 minutes. When I call the proposers back into the room, we'll discuss the experiment.

Do not talk to other people in the room and do not look at other people's forms or show your form to anyone else. In this experiment, we are going to use candy for money. As the proposer, you will determine how \$10 (or 10 pieces of candy) will be distributed between you and an anonymous responder. The money (candy) will be put into your coded envelope to be collected after the experiment. When I tell you to turn over your form, this is what you will do:

Enter the code from your ticket note at the top of the form, and then put your ticket away. You will need to turn in your ticket to claim your money (candy).

As the proposer, you will choose how to distribute the money (candy) on the Proposal Selection Form. Then, the form will be presented to the (anonymous) responder. The responder will look at the proposal and decide whether or not to accept. If the responder accepts, by circling ACCEPT on the form, you will each receive the amount circled in the proposal. If the responder rejects the proposal, by circling REJECT, both the responder AND you will receive nothing.

If there are no questions, complete your portion of the form.

Do not talk to other people in the room. When I hand you a form, leave it face down on the desk. Do not look at other people's forms or show yours to anyone else. In this experiment, we are going to use candy for money. As the responder, you will accept or reject a proposal for the distribution of \$10 (or 10 pieces of candy) between you and an anonymous proposer.

The money (candy) will be put into your coded envelope to be collected after the experiment. When I tell you to turn over the form, this is what you will do:

Write the code from your ticket on the line for the responder's code. Put your ticket away; you will need it later to claim the envelope with your money (candy) in it."

Read the proposal at the top of the form. Transfer the proposal information to the bottom half of the form by filling in the blanks.

Decide whether you want to Accept or Reject the proposal. Circle your response. Turn the form over and hand it to me face down. Sit quietly until everyone is finished.

Now that the experiment is over and I have collected the forms I will distribute the payoffs on a candy board. I will call you up by rows or sections.

Come up even if you rejected the offer.

**Predictions:**

1. Predict the outcome of ultimatum games: What do you think is the most common proposal? What types of proposals are most commonly accepted? Rejected?
2. How do you think people decide which proposal to circle?
3. What factors determine whether to accept or reject a proposal?
4. Use the diagram to determine what would be the "rational" decision of the responder.

		Responder	
		Accept	Reject
Proposer	High Offer	$10-x$ $0+x$	$0$ $0$
	Low Offer	$10-x$ $0+x$	$0$ $0$

Outcomes:

### Surprising Results

Researchers' continuing fascination with ultimatum games derives from the fact that players do not act as the economic model of self-interest predicts. Surprisingly, proposers generally offer more than the minimum, and even more surprisingly, responders frequently reject low offers, choosing to receive nothing. Hundreds of ultimatum games conducted by scores of researchers have produced the following results:

- The mean split is 60% / 40% (meaning that in a \$10 game, the proposer offers the responder \$4 and keeps \$6).
- The modal (most common) offer is a 50%-50% split.
- Approximately 20% of low offers are rejected.

The results, especially the rejection of low offers by responders, pose a serious challenge to the selfishness axiom. The behavior of proposers could still be seen as materially self-interested, but only in the unlikely scenario that the proposer assumes that other people do not have the wealth maximizing motivation that he does. In that scenario, a proposer who fears that the responder will not act in his self-interest and accept any offer over \$0 will make a more generous offer. This serves his self-interest by increasing the chance that the offer will be accepted. However, while a seemingly generous offer can be explained as self-interested, the explanation doesn't work for responders who reject offers knowing that they will end up with nothing.

Questions:

1. Are you surprised by the results? Why?
2. Would the proposer's strategy change if rules of the game were changed to what you see below:
  - A. Responders will have the option to select the best proposal offered by multiple players. And if a proposal is not selected than the proposer will not receive any candy.
  - B. What is there were multiple rounds (repeating games)
3. How does one's sense of fairness impact trade?
4. In what situations will trade be made regardless of fairness?
5. Does free trade promote or prevent selfishness? Explain?
6. Do you feel that this is a good representation of Free trade? Why or why not?
7. Create a theory to explain why we see more split offers in societies with free markets.





## The Prisoner's Dilemma

**Story:** Two Prisoner's are accused of robbing a bank together. They are in different integration rooms and the police propose the following choices to both prisoners. They can confess and rat out their partner or they can remain silent.

If they both remain silent the police have enough evidence to put them both in jail for 1 year.

If they both confess they will get 5 years each.

If one confesses and the other stays quiet, the one who confesses goes home that night and other goes to jail for 10 years.

Using the Matrix below predict what will happen. Explain your answer.

		Prisoner 2	
		Confess	Stay Quiet
Prisoner 1	Confess	5 years, 5 years	0 years, 10 years
	Stay Quiet	10 years, 0 years	1 year, 1 year

**Thoughts to consider about the story:**

1. What are the payoffs?
2. What does prisoner one want? What does Prisoner two want? (You have to know what you want before you can get it).
3. Do both know the rules?
4. If you eliminate strictly dominated strategies what are you left with?

Assuming the number one priority for both prisoner's to get out of jail as soon as possible how many years will they be staying in jail? Explain your logic.

### The Experiment: The Grade Game

You and your group will be partnered with another group. You will have a decision to make regarding your grades. Below is the diagram of the game; including strategies and payoffs.

		Group 2	
		Alpha	Beta
Group 1	Alpha	B- (80), B- (80)	A+ (100), C (75)
	Beta	C (75), A+ (100)	B+ (88), B+ (88)

Your grade for the assignment will be dependent upon what you and the other group picks.

#### Thoughts to consider during the experiment:

1. What should you pick? What do you want? What does the other group want?
2. Is there a dominated strategy?
3. What should you pick after you have spoken to the other group and received their assurance that they will pick beta?
4. Does it make a difference if this is a repeating game? Why or why not?

#### Thoughts to consider after the experiment:

1. What are some lessons of this game?
2. What does this game say about self-interest?
3. How does trust factor in when making a decision?
4. Is reputation a strong enough method to restrict selfishness? Explain why or why not?
5. Does it matter if it is a repeating game?
6. Should the government enforce cooperation and if so how does it become a new prisoner's dilemma?

7. What happens during the last game? Relate to a politician and their last term or doing business with someone for the last time.
8. Why didn't I tell you how many rounds?
9. If we played again and I had you pick the group to partner with would you pick the same group?
10. How would you have played the game if the teacher said that the next round we play you can select a group to partner with, if you and your group are not picked you and your group fail?
11. What does this game say about capitalism if there is no repeating game? If there is repeating games?
12. Use the basic matrix below to help you relate this experiment to the following situations:

		Player 2	
		Defect (self-interest)	Cooperate
Player 1	Defect (self-interest)	Bad, Bad	Best, Worst
	Cooperate	Worst, Best	Good, Good (best collectively )

- a. During the Cold War the decision by both the USA and USSR to arm or disarm.
- b. Two competing companies and their decision to lower prices or maintain prices.
- c. USA and China decided whether to lower co2 emissions or maintain current levels.
- d. An Athletes decision to use steroids or not.
- e. Members of the same political party decided to mudslinging or not in primaries.

If picking dominated strategies is the best for all parties then what are some ways to better ensure that dominated strategies will be selected?

Consider the following:

- A. Government
- B. Reputation
- C. Repeating games
- D. Self Interest
- E. Choice

