Sec 5.6 p. 407 1-25 odd

HWK

$uv - \int v\,du$

#1) $\int x e^{2x}\,dx$ $u = x$, $du = dx$, $dv = e^{2x}dx$, $v = \frac{1}{2}e^{2x}$

$\frac{1}{2}\int e^{2x}dx$, $u = 2x$, $du = 2dx$, e^{u}, $\frac{1}{2}e^{2x}$

$= \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}dx$

$= \frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}dx$

$\boxed{= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C}$

#3) $\int x\sin 4x\,dx$ $u = x$, $du = dx$, $dv = \sin 4x\,dx$, $v = -\frac{1}{4}\cos 4x$

$\int \sin 4x$, $u = 4x$, $\frac{1}{4}\int \sin u$, $-\frac{1}{4}\cos 4x$

$= -\frac{1}{4}x\cos 4x + \frac{1}{4}\int \cos 4x\,dx$

$\int \cos 4x$, $u = 4x$, $\frac{1}{4}\int \cos u$, $\frac{1}{4}\sin 4x$

$= \boxed{-\frac{1}{4}x\cos 4x + \frac{1}{16}\sin 4x + C}$

#5. $\int x^2\cos 3x\,dx$ $u = x^2$, $du = 2x\,dx$, $dv = \cos 3x\,dx$, $v = \frac{1}{3}\sin 3x$

$= \frac{1}{3}x^2\sin 3x - \frac{1}{3}\int \sin 3x\,(2x\,dx)$

$= \frac{1}{3}x^2\sin 3x - \frac{2}{3}\int x\sin 3x\,dx$ $U = x$, $du = dx$, $dv = \sin 3x\,dx$, $v = -\frac{1}{3}\cos 3x$

$= \frac{1}{3}x^2\sin 3x - \frac{2}{3}\left[-\frac{1}{3}x\cos 3x + \frac{1}{3}\int \cos 3x\,dx\right]$

$= \frac{1}{3}x^2\sin 3x - \frac{2}{3}\left[-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right] + C$

$\boxed{= \frac{1}{3}x^2\sin 3x + \frac{2}{9}x\cos 3x - \frac{2}{27}\sin 3x + C}$

#7) $\int (\ln x)^2 dx$ $u = (\ln x)^2$, $du = 2(\ln x)\cdot\frac{1}{x}dx$, $dv = dx$, $v = x$

$= x(\ln x)^2 - 2\int x\cdot\frac{1}{x}(\ln x)dx$

$= x(\ln x)^2 - 2\int \ln x\,dx$ $u = \ln x$, $du = \frac{1}{x}dx$, $dv = dx$, $v = x$

$= x(\ln x)^2 - 2\left[x\ln x - \int \underbrace{x\cdot\frac{1}{x}dx}_{1\,dx}\right]$

$= x(\ln x)^2 - 2[x\ln x - x] + C$

$= x(\ln x)^2 - 2x\ln x + 2x + C$ where $C = -2C_1$

Mnemonic Device

LIATE w/ is helpful for selecting u

- **L**ogarithmic
- **I**nverse trig
- **A**lgebraic
- **T**rigonometric
- **E**xponential function

If integrand has several factors, then try to choose "u" which appears as high as possible on the list.

Ex xe^{2x} Alg · Exp
Since A appears before E
choose $u = x$

Sometimes integration turns out to be similar regardless of u and dv, refer to LIATE when in doubt

$\sin 2x = 2\sin x\cos x$

#9) $2\int \theta \sin\theta\cos\theta\, d\theta$ $\qquad u=\theta \quad dv=\sin 2\theta\, d\theta$

$\frac{1}{2}\int \theta\, 2\sin\theta\cos\theta\, d\theta \qquad du=d\theta \quad v=\frac{1}{2}\cos 2\theta$

$\frac{1}{2}\int \theta\sin 2\theta\, d\theta \qquad uv-\int v\,du$

$\boxed{\frac{1}{4}}\int \theta\sin 2\theta\, d\theta = \frac{1}{2}\theta\cos 2\theta - \frac{1}{2}\int\cos 2\theta\, d\theta$

$= \frac{1}{2}\theta\cos 2\theta - \frac{1}{2}\cdot\frac{1}{2}\sin 2\theta + C$

$= \frac{1}{2}\left[\frac{1}{2}\theta\cos 2\theta - \frac{1}{4}\sin 2\theta + C\right]$

$= \frac{1}{4}\left[\theta\cos 2\theta - \frac{1}{2}\sin 2\theta + C\right]$

#11) $\int t^2 \ln t\, dt \qquad u=\ln t \quad dv=t^2\, dt$

$du=\frac{1}{t}dt \quad v=\frac{1}{3}t^3$

$= \frac{1}{3}t^3\ln t - \frac{1}{3}\int t^3\cdot\frac{1}{t}\, dt$

$= \frac{1}{3}t^3\ln t - \frac{1}{3}\int t^2\, dt$

$= \boxed{\frac{1}{3}t^3\ln t - \frac{1}{9}t^3 + C}$ or $\frac{1}{9}t^3(3\ln t - 1) + C$

#13) $\int e^{2\theta}\sin 3\theta\, d\theta \qquad u=\sin 3\theta \quad dv=e^{2\theta}\, d\theta$

$du = 3\cos 3\theta\, d\theta \quad v=\frac{1}{2}e^{2\theta}$

$uv - \int v\, du$

$= \frac{1}{2}e^{2\theta}\sin 3\theta - \frac{3}{2}\int e^{2\theta}\cos 3\theta\, d\theta \qquad u=\cos 3\theta \quad dv=e^{2\theta}\, d\theta$

$du=-3\sin 3\theta\, d\theta \quad v=\frac{1}{2}e^{2\theta}$

$= \frac{1}{2}e^{2\theta}\sin 3\theta - \frac{3}{2}\left[\frac{1}{2}e^{2\theta}\cos 3\theta + \frac{3}{2}\int e^{2\theta}\sin 3\theta\, d\theta\right]$

So;

$1\int e^{2\theta}\sin 3\theta\, d\theta = \frac{1}{2}e^{2\theta}\sin 3\theta - \frac{3}{4}e^{2\theta}\cos 3\theta - \frac{9}{4}\int e^{2\theta}\sin 3\theta\, d\theta$

$+\frac{9}{4}$

$\frac{13}{4}\int e^{2\theta}\sin 3\theta\, d\theta = \frac{1}{2}e^{2\theta}\sin 3\theta - \frac{3}{4}e^{2\theta}\cos 3\theta$

$\int e^{2\theta}\sin 3\theta\, d\theta = \frac{4}{13}\left[\frac{1}{2}e^{2\theta}\sin 3\theta - \frac{3}{4}e^{2\theta}\cos 3\theta\right] + C$

$= \boxed{\frac{1}{13}e^{2\theta}\left[2\sin 3\theta - 3\cos 3\theta\right] + C}$

$C = \frac{4}{13}C_1$

#15) $\int_0^1 te^{-t}dt$ $\quad u=t \quad du=dt \quad dv=e^{-t}dt \quad v=-e^{-t}$ $\quad u=-t \quad du=t$

Log
A
Inverse
Trig
e

$uv-\int v\,du$

$\int_0^1 te^{-t}dt = -te^{-t} + \int e^{-t}dt$

$= -te^{-t}\Big]_0^1 + \left[-e^{-t}\right]_0^1$

$= \left[-1e^{-1}-0\right] + \left[-e^{-1}-(-1)\right]$

$= \frac{-1}{e} - \frac{1}{e} + 1$

$= \boxed{1-\frac{2}{e}}$

#17) $\int_0^{\pi/2} x\cos 2x\,dx$ $\quad u=x \quad du=dx \quad dv=\cos 2x\,dx \quad v=\frac{1}{2}\sin 2x$ $\quad u=2x$

$\int_0^{\pi/2} x\cos 2x\,dx = \frac{1}{2}x\sin 2x\Big|_0^{\pi/2} - \frac{1}{2}\int \sin 2x\,dx$ $\quad \frac{1}{2}\cdot -\frac{1}{2}\cos 2x$

$= \frac{1}{2}x\sin 2x\Big|_0^{\pi/2} + \frac{1}{4}\cos 2x\Big|_0^{\pi/2}$

$\int x$

$= \left[\frac{\pi}{4}\sin 2(\frac{\pi}{2}) - 0\right] + \left[\frac{1}{4}\cos 2(\frac{\pi}{2}) - \frac{1}{4}\cos 0\right]$

(-1,0)

$= \left[\frac{\pi}{4}\sin\pi\right] + \left[\frac{1}{4}\cos\pi - \frac{1}{4}\cos 0\right]$

$= 0 + \frac{1}{4}(-1) - \frac{1}{4}(1)$

$= -\frac{1}{4}-\frac{1}{4} = -\frac{2}{4} = \boxed{-\frac{1}{2}}$

#19) $\int_0^{1/2} \sin^{-1}x\,dx$ $\quad u=\sin^{-1}x \quad du=\frac{1}{\sqrt{1-x^2}}dx \quad dv=dx \quad v=x$

$uv-\int v\,du$

$\int_0^{\pi/2} \sin^{-1}x\,dx = x\sin^{-1}x\Big|_0^{1/2} - \int x\cdot\frac{1}{\sqrt{1-x^2}}dx$ $\quad U=1-x^2 \quad dU=-2x\,dx$

$= x\sin^{-1}x\Big|_0^{1/2} + \frac{1}{2}\int u^{-1/2}$ $\quad \int_0^{1/2} \to \int_1^{3/4}$ $\quad 1-\frac{1}{4}$

$= \left[\frac{1}{2}\sin^{-1}\frac{1}{2} - 0\right] + \frac{1}{2}\left[2u^{1/2}\right]_1^{3/4}$

$= \left[\frac{1}{2}\cdot\frac{\pi}{6} - 0\right] + \left[(\frac{3}{4})^{1/2} - (1)^{1/2}\right]$

$= \boxed{\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1}$

$1\ [\ \ -12\ +\ln\sqrt{2})$

#21) $\int_1^4 \ln\sqrt{x}\,dx = \int_1^4 \frac{1}{2}\ln x\,dx$

$\frac{1}{2}\int_1^4 \ln x\,dx$ $\quad u=\ln x \quad dv=dx$ $\quad du=\frac{1}{x}dx \quad v=x$

$\frac{1}{2}\int_1^4 \ln x\,dx = \frac{1}{2}(x\ln x) - \int_1^4 x\cdot\frac{1}{x}dx$

$= \frac{1}{2}[x\ln x]_1^4 - \int_1^4 1\,dx$

$= \frac{1}{2}[x\ln x]_1^4 - x\Big|_1^4$

$= \frac{1}{2}[4\ln 4 - 1\cdot\ln 1] - [4-1]$

$= \frac{1}{2}[4\ln 4 - 0] - [3]$

$= \boxed{2\ln 4 - \frac{3}{2}}$

#~~23~~) $\int_0^1 (x^2-1)e^x\,dx$ $\quad u=x^2-1 \quad dv=e^x dx$ $\quad du=2x\,dx \quad v=e^x$

$\int_0^1 (x^2-1)e^x dx = e^x(x^2-1)\Big|_0^1 - 2\int x e^x dx$ $\quad U=x \quad dv=e^x dx$ $\quad du=dx \quad v=e^x$

$= e^x(x^2-1)\Big|_0^1 - 2\left[xe^x - \int e^x dx\right]$

$= e^x(x^2-1)\Big|_0^1 - 2[xe^x - e^x]_0^1$

$= [e^1(1-1) - e^0(0-1)] - 2[(1e^1 - e^1) - (0 - e^0)]$

$1(-1)$

$= [0+1] - 2[0+1]$

$= 1-2 =$

$= \boxed{-1}$

#25) $\int \sin\sqrt{x}\,dx$

$\int \sin w \cdot 2w\,dw$

$w=\sqrt{x}$
$w^2=x$
$x=w^2$
$dx=2w\,dw$

Now, by Parts

$\int \sin w\cdot 2w\,dw$ $\quad u=2w \quad dv=\sin w\,dw$ $\quad du=2dw \quad v=-\cos w$

$= -2w\cos w + 2\int \cos w\,dw$

$= -2w\cos w + 2\sin w + C$

$\boxed{-2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C}$

p407 #4–20 evens

4) $\int x \ln x \, dx$ $u = \ln x$ $dv = x\,dx$

$uv - \int v\,du$ $du = \frac{1}{x}dx$ $v = \frac{x^2}{2}$

$\frac{1}{2}x^2 \ln x - \frac{1}{2}\int \frac{1}{\cancel{x}} \cdot x^{\cancel{2}} dx$

$\boxed{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C}$

6) $\int x^2 \sin 2x \, dx$ $u = x^2$ $dv = \sin 2x\,dx$

$du = 2x\,dx$ $v = -\frac{1}{2}\cos 2x$

$-\frac{1}{2}x^2 \cos 2x + \frac{1}{\cancel{2}} \cdot \cancel{2} \int x \cos 2x \, dx$ $u = x$ $dv = \cos 2x\,dx$

$du = dx$ $v = \frac{1}{2}\sin 2x$

$uv - \int v\,du$

$-\frac{1}{2}x^2 \cos 2x + \frac{1}{2}x \sin 2x - \frac{1}{2}\int \sin 2x \, dx$

$\boxed{-\frac{1}{2}x^2 \cos 2x + \frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + C}$

8) $\int \sin^{-1} x \, dx$ $u = \sin^{-1} x$ $dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}dx$ $V = x$

$x \sin^{-1} x + \frac{1}{2}\int \frac{x}{\sqrt{1-x^2}}dx$

$u^{-1/2}$

$u = 1 - x^2$

$\boxed{x \sin^{-1} x + \sqrt{1-x^2} + C}$ $du = -2x\,dx$

10) $\int \theta \sec^2\theta \, d\theta$

$u = \theta \qquad dv = \sec^2\theta \, d\theta$

$du = d\theta \qquad v = \tan\theta$

$\theta\tan\theta - \int \tan\theta \, d\theta$

$\theta\tan\theta - \int \frac{\sin\theta}{\cos\theta} d\theta \qquad u = \cos\theta$

$+\int \frac{1}{u} du \qquad du = -\sin\theta \, d\theta$

$\theta\tan\theta + \ln|\cos\theta| + C$

12) $\int t^3 e^t \, dt$

$u = t^3 \qquad dv = e^t dt$

$du = 3t^2 dt \qquad v = e^t$

$3\int t^2 e^t dt$

$t^3e^t - 3[t^2e^t - 2\int te^t dt] \qquad u = t^2 \qquad dv = e^t dt$

$\downarrow \qquad du = 2t\,dt \quad v = e^t$

$t^3e^t - 3t^2e^t + 6\int te^t dt \qquad u = t \quad dv = e^t dt$

$du = dt \quad v = e^t$

$t^3e^t - 3t^2e^t + 6[te^t - \int e^t dt]$

$\boxed{t^3e^t - 3t^2e^t + 6te^t - 6e^t + C}$

14) $\int e^{-\theta}\cos 3\theta \, d\theta$

$u = \cos 3\theta \qquad dv = \int e^{-\theta} d\theta \quad u = -\theta \quad du = d\theta$

$du = -3\sin 3\theta \, d\theta \qquad v = -e^{-\theta}$

$-e^{-\theta}\cos 3\theta - 3\int e^{-\theta}\sin 3\theta \, d\theta$

$u = \sin 3\theta \qquad dv = e^{-\theta} d\theta$

$du = 3\cos 3\theta \, d\theta \qquad v = -e^{-\theta}$

$\int e^{-\theta}\cos\theta \, d\theta = -e^{\theta}\cos 3\theta - 3[-e^{-\theta}\sin 3\theta + 3\int e^{-\theta}\cos 3\theta \, d\theta]$

$\int e^{-\theta}\cos\theta \, d\theta = -e^{\theta}\cos 3\theta + 3e^{-\theta}\sin 3\theta - 9\int e^{-\theta}\cos 3\theta \, d\theta$

$+9\int e^{-\theta}\cos\theta \, d\theta \qquad +9\int e^{-\theta}\cos 3\theta \, d\theta$

$10\int e^{-\theta}\cos\theta \, d\theta = \boxed{\frac{-e^{\theta}\cos 3\theta + 3e^{-\theta}\sin 3\theta}{10} + C}$

16) $\int_1^4 \sqrt{t}\ln t\,dt$

$u = \ln t$ $\quad dv = \sqrt{t}\,dt$ ($\int t^{1/2}$)

$du = \frac{1}{t}\,dt$ $\quad v = \frac{2}{3}t^{3/2}$

$\frac{2}{3}t^{3/2}\ln t\Big|_1^4 - \frac{2}{3}\int \frac{1}{t}\cdot t^{3/2}\,dt$

$\left(\frac{2}{3}\cdot 8\cdot \ln 4 - 0\right) - \frac{2}{3}\int_1^4 t^{1/2}\,dt$

$\frac{16}{3}\ln 4 - \frac{2}{3}\left[\frac{2}{3}t^{3/2}\right]_1^4$

$\frac{16}{3}\ln 4 - \frac{4}{9}\left[\overset{7}{8-1}\right]$

$\boxed{16/3 \ln 4 - 28/9}$

18) $\int_0^1 x^2 e^{-x}\,dx$

$u = x^2$ $\quad dv = e^{-x}\,dx$

$du = 2x\,dx$ $\quad v = -e^{-x}$

$-e^{-x}x^2\Big|_0^1 + 2\int_0^1 x e^{-x}\,dx$ $\quad u = x$ $\quad dv = e^{-x}\,dx$

$du = dx$ $\quad v = -e^{-x}$

$[-e^{-1} - 0] + 2\left[-xe^{-x}\Big|_0^1 + \int_0^1 e^{-x}\,dx\right]$

$-e^{-1} + 2\left[-e^{-1} + \int_0^1 e^{-x}\,dx\right]$

$-e^{-1} - 2e^{-1} + 2\left[-e^{-x}\right]_0^1$

$-1e^{-1} - 2e^{-1} + 2[-e^{-1} + 1]$

$-1e^{-1} - 2e^{-1} - 2e^{-1} + 2 = \boxed{2 - 5e^{-1}}$

20) $\int_{\pi/4}^{\pi/2} x\csc^2 x\,dx$ $\quad u = x \quad dv = \csc^2 x\,dx$

$\quad du = dx \quad v = -\cot x$

$-x\cot x \Big|_{\pi/4}^{\pi/2} + \int_{\pi/4}^{\pi/2} \cot x\,dx$

(0,1)

$\left[-\frac{\pi}{2}\cdot 0 + \pi/4(1)\right] + \int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x}\,dx$ $\quad u = \sin x$

$\quad du = \cos x\,dx$

$0 + \pi/4 \qquad \int_{\pi/4}^{\pi/2} \frac{1}{u}\,du$

$\ln|u| \Big]_{\pi/4}^{\pi/2}$

$\pi/4 + \left[\ln|u|\right]_{\pi/4}^{\pi/2}$

$\pi/4 + \ln|\sin x| \Big|_{\pi/4}^{\pi/2}$

0,1

$\pi/4 + [\ln\sin\pi/2 - \ln\sin\pi/4]$

$\pi/4 + \ln 1 - \ln\frac{1}{\sqrt{2}}$

$\pi/4 + 0 - \ln 2^{-1/2}$

$\boxed{\pi/4 + \frac{1}{2}\ln 2}$