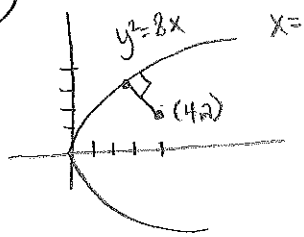
max/min word problems

3) Find min distance from (4,2) to parabola $y^2=8x$



$x=\frac{y^2}{8}$

$d=\sqrt{(x-4)^2+(y-2)^2}$

$d^2=(\frac{y^2}{8}-4)^2+(y-2)^2$

$(d^2)'=2(\frac{y^2}{8}-4)(\frac{1}{4}y)+2(y-2)$

$=\frac{1}{2}y(\frac{y^2}{8}-4)+2y-4$

$=\frac{y^3}{16}-2y+2y-4=0$

$\frac{y^3}{16}-4=0$

$\frac{y^3}{16}=4$

$y^3=\sqrt[3]{64}$

$y=4$ so, $x=\frac{4^2}{8}$

$x=2$

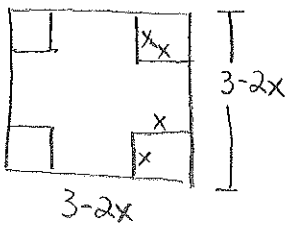
min distance: plug in

$d=\sqrt{(2-4)^2+(4-2)^2}$

$d=\sqrt{(-2)^2+(2)^2}$

$=\sqrt{4+4}=\sqrt{8}=$ **$2\sqrt{2}$ units**

#3)



② $V=lwh$

$V=(3-2x)(3-2x)(x)$

$V=9x-12x^2+4x^3$

Domain: $0\le x\le 3/2$

③ $V'=9-24x+12x^2=0$

$3(3-8x+4x^2)=0$

$(3-2x)(1-2x)=0$

$x=\cancel{3/2}, 1/2$

↑ guess

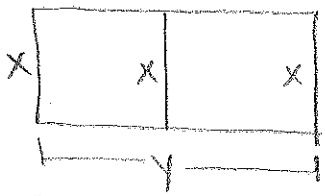
$V=(3-2(\frac{1}{2}))(3-2(\frac{1}{2}))(\frac{1}{2})$

$V=(2)(2)(\frac{1}{2})$

$V=2$ ft^3

Dimensions: $\frac{1}{2}\times 2\times 2$

#4) Rectangular pasture w/ 2 corals from 800 yds of fencing. Find dimensions w/ max area



② $A=lw$

$A=x(y)$

$A=x(\frac{800-3x}{2})$

$A=400x-\frac{3}{2}x^2$

$800=3x+2y$

$2y=800-3x$

$y=\frac{800-3x}{2}$

③ $A'=400-3x=0$

$-3x=-400$

$x=\frac{400}{3}$ So, $y=\frac{800-3(\frac{400}{3})}{2}=\frac{800-400}{2}$

$y=200$

Dimensions: $\frac{400}{3}\times 200$

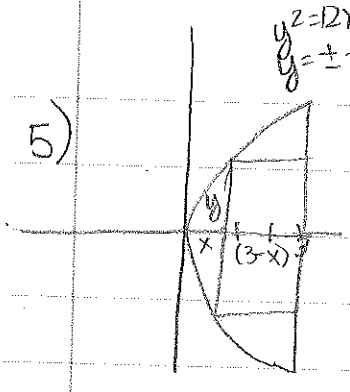
area: $80000/3$ yd^2

5)

$y^2 = 12x$

$y = \pm\sqrt{12x}$

$x = \frac{y^2}{12}$



$A = lw$

$A = 2y(3-x)$

$A = 2y\left(3 - \frac{y^2}{12}\right)$

$A = 6y - \frac{1}{6}y^3$

$A' = 6 - \frac{1}{2}y^2 = 0$

$\frac{1}{2}y^2 = 6$

$y^2 = 12$

$y = \sqrt{12}$ or $2\sqrt{3}$

$x = \frac{(2\sqrt{3})^2}{12} = \frac{12}{12} = 1$

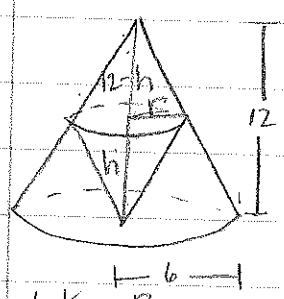
So $x = 1$ $(3-x) = 2$

Dimensions:

$4\sqrt{3} \times 2$

---

6)



$V = \frac{1}{3}\pi r^2 h$

$V = \frac{\pi}{3} r^2 (12 - 2r)$

$V = 4\pi r^2 - \frac{2}{3}\pi r^3$

$\frac{1}{2}\frac{6}{12} = \frac{r}{12-h}$

$2r = 12 - h$

$h = 12 - 2r$

$V' = 8\pi r - 2\pi r^2 = 0$

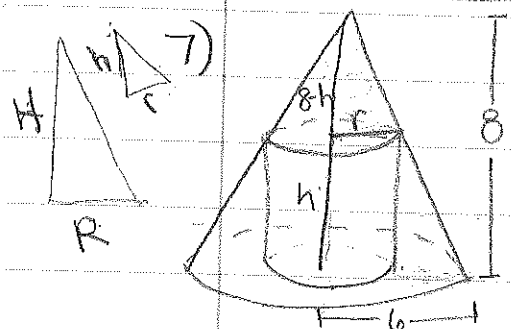
$2\pi r(4 - r) = 0$

$r = 4$ $\quad h = 12 - 2(4) = 4$

So, $r = 4$, $h = 4$, $V = \frac{64}{3}\pi$

---

7)



$V = \pi r^2 h$

$V = \pi r^2 \left(8 - \frac{4}{3}r\right)$

$V = 8\pi r^2 - \frac{4}{3}\pi r^3$

$\frac{3}{4}\frac{6}{8} = \frac{r}{8-h}$

$4r = 24 - 3h$

$3h = 24 - 4r$

$h = 8 - \frac{4}{3}r$

$V' = 16\pi r - 4\pi r^2 = 0$

$4\pi r(4 - r) = 0$

$r = 4$ $\quad h = 8 - \frac{4}{3}(4) = 8 - \frac{16}{3} \quad \frac{24}{3} - \frac{16}{3} = \frac{8}{3}$

$r = 4''$ $\quad h = \frac{8}{3}''$