Ch. 3 Review
True / False
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True 1) $\frac{d}{dx}[f(x)+g(x)] = f'(x)+g'(x)$

False 2) $\frac{d}{dx}[f(x)\cdot g(x)] = f'(x)\cdot g'(x)$

Proof:

$$= \lim_{h\to 0} \frac{f(x+h)g(x+h)-f(x)g(x)}{h}$$

"Magic"

$$= \lim_{h\to 0} \frac{f(x+h)g(x+h)-f(x+h)g(x)+f(x+h)g(x)-f(x)g(x)}{h}$$

$$= \lim_{h\to 0} f(x+h)\left[\frac{g(x+h)-g(x)}{h}\right] + \lim_{h\to 0} g(x)\left[\frac{f(x+h)-f(x)}{h}\right]$$

$$= f(x)\cdot g'(x) + g(x)\cdot f'(x)$$

True 3) $\frac{d}{dx}[f(g(x))] = f'(g(x))\, g'(x)$

True 4) $\frac{d}{dx}\sqrt{f(x)} = \frac{1}{2}(f(x))^{-1/2}\cdot f'(x)$

$$= \frac{f'(x)}{2\sqrt{f(x)}}$$

False 5) $\frac{d}{dx} f(\sqrt{x}) = f'(\sqrt{x})\,\frac{1}{2}x^{-1/2}$

$$= \frac{f'(\sqrt{x})}{2\sqrt{x}} \neq \frac{f'(x)}{2\sqrt{x}}$$

False 6) If $y = e^2$

$y' = 0$ not $y' = 2e$

False 7) $\frac{d}{dx}(10^x) = 10^x \cdot \ln 10$

not $x10^{x-1}$

False 8) $\frac{d}{dx}(\ln 10) = 0$

not $\frac{1}{10}$

True 9) $\frac{d}{dx}(\tan^2 x) = \frac{d}{dx}(\sec^2 x)$

$\frac{d}{dx}(\tan x)^2 = \frac{d}{dx}(\sec x)^2$

$2\tan x(\sec^2 x) = 2\sec x(\sec x \tan x)$

$2\tan x \sec^2 x = 2\tan x \sec^2 x$

False 10) $\frac{d}{dx}|x^2+x| = |2x+1|$

$= 2x+1$

True 11) $g(x) = x^5$, then

alt aler form $\lim_{x\to 2} \frac{g(x)-g(2)}{x-2} = 80$

$\Downarrow$

$g(x) = x^5$

$g'(x) = 5x^4$

$g'(2) = 5\cdot 2^4$

$= 5\cdot 16$

$= 80$

Ch. 3 Review

p261 1-29 odd

I. Calculate y':

1.) $y=(x+2)^8(x+3)^6$

$y'=(x+2)^8\cdot 6(x+3)^5+(x+3)^6\,8(x+2)^7$

$=2(x+2)^7(x+3)^5\left[\overset{3x+6+4x+12}{3(x+2)+4(x+3)}\right]$

$\boxed{=2(x+2)^7(x+3)^5(7x+18)}$

3.) $y=\dfrac{x}{\sqrt{9-4x}}$

$y'=\dfrac{\sqrt{9-4x}\,(1)-x\,\frac{1}{2}(9-4x)^{-1/2}(-4)}{(9-4x)}$

$=\dfrac{(9-4x)^{1/2}+2x(9-4x)^{-1/2}}{(9-4x)}$

$=\dfrac{(9-4x)^{-1/2}\left[(9-4x)+2x\right]}{(9-4x)}$

$\boxed{=(9-4x)^{-3/2}(9-2x)}$

5.) $y=\sin(\cos x)$

$y'=\cos(\cos x)\cdot -\sin x$

$\boxed{=-\sin x\cos(\cos x)}$

7.) $y=xe^{-1/x}$ $\quad -x^{-1}$

$y'=x\cdot e^{-1/x}\cdot x^{-2}+e^{-1/x}(1)$

$\boxed{=e^{-1/x}\left(\frac{1}{x}+1\right)}$

9.) $y=\tan\sqrt{1-x}$

$y'=\sec^2\sqrt{1-x}\cdot\left(\frac{1}{2}(1-x)^{-1/2}\cdot -1\right)$

$\boxed{=\dfrac{-\sec^2\sqrt{1-x}}{2\sqrt{1-x}}}$

11.) $y=\dfrac{x}{8-3x}$

$y'=\dfrac{(8-3x)(1)-x(-3)}{(8-3x)^2}$

$=\dfrac{8-\cancel{3x}+\cancel{3x}}{(8-3x)^2}\boxed{=\dfrac{8}{(8-3x)^2}}$

13.) $y=e^{cx}(c\sin x-\cos x)$

$y'=e^{cx}(c\cdot\cos x+\sin x)+ce^{cx}(c\sin x-\cos x)$

$=e^{cx}(c\cdot\cancel{\cos x}+\sin x+c^2\sin x-c\cdot\cancel{\cos x})$

$=e^{cx}(c^2\sin x+\sin x)$

$\boxed{=e^{cx}\sin x\,(c^2+1)}$

15) $y=e^{e^x}$

$y'=e^{e^x}\cdot e^x$

$\boxed{=e^{e^x+x}}$

17.) $x^2y^3+3y^2=x-4y$

$x^2\cdot 3y^2\cdot y'+y^3\cdot 2x+6y\cdot y'=1-4y'$

$y'(3x^2y^2+6y+4)=1-2xy^3$

$\boxed{y'=\dfrac{1-2xy^3}{3x^2y^2+6y+4}}$

19.) $y=\log_{10}(x^2-x)$

$y'=\dfrac{1}{(x^2-x)\ln 10}(2x-1)$

$\boxed{=\dfrac{2x-1}{(x^2-x)\ln 10}}$

21) $y=\ln\sin x-\frac{1}{2}\sin^2x$

$y'=\dfrac{1}{\sin x}\cdot\cos x-\frac{1}{2}\left[2\sin x\cos x\right]$

$\boxed{=\cot x-\sin x\cos x}$

23.) $y=\sin\left(\tan\sqrt{1+x^3}\right)$

$y'=\cos\left(\tan\sqrt{1+x^3}\right)\cdot\sec^2\sqrt{1+x^3}\cdot\frac{1}{2}(1+x^3)^{-1/2}\cdot 3x^2$

$\boxed{=\dfrac{3x^2\cos\left(\tan\sqrt{1+x^3}\right)\sec^2\sqrt{1+x^3}}{2\sqrt{1+x^3}}}$

25) $y = \dfrac{\sqrt{x+1}\,(2-x)^5}{(x+3)^7}$

$\ln y = \frac{1}{2}\ln(x+1) + 5\ln(2-x) - 7\ln(x+3)$

$\frac{1}{y}\cdot y' = \frac{1}{2}\cdot\frac{1}{x+1} + \frac{5}{2-x}(-1) - \frac{7}{x+3}$

$\boxed{y' = \left[\dfrac{\sqrt{x+1}\,(2-x)^5}{(x+3)^7}\right]\left[\dfrac{1}{2(x+1)} - \dfrac{5}{2-x} - \dfrac{7}{x+3}\right]}$

27) Find $f''(0)$, If

$f(x) = \frac{1}{(2x-1)^5}$

$f(x) = (2x-1)^{-5}$

$f'(x) = -5(2x-1)^{-6}(2)$

$= -10(2x-1)^{-6}$

$f''(x) = 60(2x-1)^{-7}(2)$

$= 120(2x-1)^{-7}$

$f''(0) = 120(2\cdot 0-1)^{-7}$

$= \frac{120}{(-1)^7} = \boxed{-120}$

29.) If $f(x) = 2^x$, find $f^{(n)}(x)$

$f'(x) = 2^x \ln 2$

$f''(x) = 2^x\cdot 0 + \ln 2\,[2^x\cdot\ln 2]$

$= 2^x[\ln 2]^2$

$f'''(x) = 2^x\cdot 0 + [\ln 2]^2\,[2^x \ln 2]$

$= 2^x[\ln 2]^3$

$\boxed{f^{(n)}(x) = 2^x[\ln 2]^n}$

31) (a) If $f(x) = x\sqrt{5-x}$ $f'(x)$

$f'(x) = x\left(\frac{1}{2}(5-x)^{-1/2}(-1)\right] + (5-x)^{1/2}(1)$

$= (5-x)^{-1/2}\left[-\frac{1}{2}x + 5 - x\right]$

$= (5-x)^{-1/2}(5 - 3/2\,x)$

$\boxed{= \dfrac{10-3x}{2\sqrt{5-x}}}$

(b) Find eq. of tan @ (1,2) and (4,4)

A) $y-2 = m(x-1)$ $f'(1) = \frac{10-3x}{2\sqrt{5-x}} = \frac{10-3}{2\sqrt{4}} = \frac{7}{4}$

$y-2 = \frac{7}{4}(x-1)$

$y-2 = \frac{7}{4}x - \frac{7}{4}$

$\boxed{y = \frac{7}{4}x + \frac{1}{4}}$

(4,4)

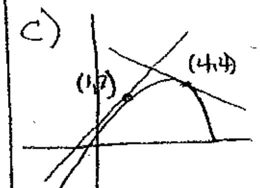
B) $y-4 = m(x-4)$ $f'(4) = \frac{10-3(4)}{2\sqrt{5-4}} = \frac{-2}{2} = -1$

$y-4 = -1(x-4)$

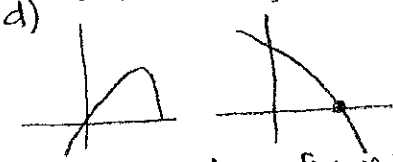
$y-4 = -x+4$

$\boxed{y = -x+8}$

c)



d) f(x) f'(x)



f' is pos where fun is Inc
f' is 0 where f gets horiz
f' is neg where f decr

35) $h(x) = f(x)g(x)$ $F(x) = f(g(x))$

$f(2)=3$
$f'(2)=-2$
$f'(5)=11$
$g(2)=5$
$g'(2)=4$

a) $h'(x) = f(x)g'(x) + g(x)f'(x)$

$= 3\cdot 4 + 5\cdot -2$

$= 12-10$

$= \boxed{2}$

b) $F'(x) = f'(g(x))g'(x)$

$= f'(5)(4)$

$= 11(4)$

$= \boxed{44}$

#36) $F(2)=1$
$f'(2)=-1$
$g(2)=4$
$g'(2)=2$
$f'(4)=3$

a) $P'(x) = f(x)g'(x) + g(x)f'(x)$

$= 1(2) + 4(-1)$

$= 2-4$

$\boxed{-2}$

b) $Q(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

$= \dfrac{4(-1) - (1)(2)}{4^2}$

$= \dfrac{-4-2}{16} = \dfrac{-6}{16} = \boxed{\dfrac{-3}{8}}$

c) $C(x) = f'(g(x))g'(x)$

$= f'(4)(2)$

$= 3(2)$

$\boxed{= 6}$

37-44, 50, 52, 57

#37) $f(x) = x^2 g(x)$

$f'(x) = x^2 \cdot g'(x) + g(x) \cdot 2x$

$= x[xg'(x) + 2g(x)]$

#38) $f(x) = g(x^2)$

$f'(x) = g'(x^2) \cdot 2x$

$= 2x\,g'(x^2)$

#39) $f(x) = [g(x)]^2$

$f'(x) = 2[g(x)] \cdot g'(x)$

#40) $f(x) = g(g(x))$

$f'(x) = g'(g(x)) \cdot g'(x)$

#41) $f(x) = g(e^x)$

$f'(x) = g'(e^x) \cdot e^x$

#42) $f(x) = e^{g(x)}$

$f'(x) = e^{g(x)} \cdot g'(x)$

#43) $f(x) = \ln|g(x)|$

$f'(x) = \frac{1}{g(x)} \cdot g'(x)$

$= \frac{g'(x)}{g(x)}$

#44) $f(x) = g(\ln x)$

$f'(x) = g'(\ln x) \cdot \frac{1}{x}$

$= \frac{g'(\ln x)}{x}$

#50) a) On what interval is $f(x) = \frac{\ln x}{x}$ increasing?

$f'(x) = \frac{x \cdot \frac{1}{x} - \ln x}{x^2}$ D: $(0, \infty)$

$= \frac{1 - \ln x}{x^2} > 0 \rightarrow$ $1 - \ln x > 0$

$-\ln x > -1$

$e^{\ln x} < e^1$ $x < e$

$\boxed{D: (0, e)}$

b) On what interval is f concave upward?

$f''(x) = \frac{x^2\left(-\frac{1}{x}\right) - (1 - \ln x) \cdot 2x}{(x^2)^2}$

$= \frac{-x - 2x + 2x\ln x}{x^4}$

$= \frac{-3x + 2x\ln x}{x^4} = \frac{x(2\ln x - 3)}{x^4}$

$= \frac{2\ln x - 3}{x^3} > 0$

$2\ln x - 3 > 0$

$e^{\ln x} > e^{\frac{3}{2}}$ $x > e^{3/2}$ ≈ 4.48 $(4.48, \infty)$

#54) $V = \frac{\pi r^2 h}{3}$

a) $dV = \frac{1}{3}\pi r^2 \cdot h'$

b) $dV = \frac{2}{3}\pi r h \cdot r'$

#57) a) Find the linearization of $f(x) = \sqrt[3]{1+3x}$ at $a=0$

$f(0) = \sqrt[3]{1+3\cdot 0}$
$= 1$

$f'(x) = \frac{1}{3}(1+3x)^{-2/3} \cdot 3$
$= (1+3x)^{-2/3}$

$f'(0) = \frac{1}{(1+3x)^{2/3}}$
$= 1$

$L(x) = f(a) + f'(a)(x-a)$
$= 1 + 1(x-0)$
$L(x) = 1 + x$

So, Give approximate value for $\sqrt[3]{1.03}$

$\sqrt[3]{1+3(.01)} = L(.01) = 1 + .01$
$\boxed{\approx 1.01}$

b) Determine values of x for which the linear approx is accurate to .1

$\sqrt[3]{1+3x} - .1 < 1+x < \sqrt[3]{1+3x} + .1$

$-.236 \qquad .40$

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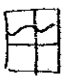

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$-.236 \qquad .40$

Ch. Review

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2) $y = \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}$

$y = x^{1/3} + x^{-1/3}$

$\boxed{y' = \frac{1}{3}x^{-2/3} - \frac{1}{3}x^{-4/3}}$

4) $y = \frac{e^x}{1+x^2}$

$y' = \frac{(1+x^2)e^x - e^x(2x)}{(1+x^2)^2}$

$= \frac{e^x(x^2-2x+1)}{(1+x^2)^2} = \boxed{\frac{e^x(x-1)^2}{(x^2+1)^2}}$

6) $y = \sin^{-1}(e^x)$

$y' = \frac{e^x}{\sqrt{1-e^{2x}}}$

8) $y = x^r e^{sx}$

$y' = x^r s e^{sx} + e^{sx} r x^{r-1}$

$= \boxed{e^{sx} x^{r-1}(sx + r)}$

10) $y = \frac{1}{\sin(x - \sin x)}$

$y' = \frac{-\cos(x-\sin x)(1-\cos x)}{[\sin(x-\sin x)]^2}$

12) $y = \ln(\csc 5x)$

$y' = \frac{-5\cancel{\csc 5x}\cot 5x}{\cancel{\csc 5x}}$

$= -5\cot 5x$

14) $y = \ln(x^2 e^x)$

$y' = \frac{x^2e^x + 2xe^x}{x^2e^x}$

$= \frac{\cancel{x}\cancel{e^x}(x+2)}{x^{\cancel{2}}\cancel{e^x}} = \boxed{\frac{x+2}{x}}$

16) $y = 5^{x\tan x}$

$y' = 5^{x\tan x}(\ln 5)(x\sec^2 x + \tan x)$

18) $x\tan y = y - 1$

$x\sec^2 y \cdot y' + \tan y = y'$

$y' - x\sec^2 y \cdot y' = \tan y$

$y'(1 - x\sec^2 y) = \tan y$

$y' = \frac{\tan y}{1 - x\sec^2 y}$

20) $y = e^{\cos x} + \cos(e^x)$

$y' = e^{\cos x} \cdot -\sin x - \sin(e^x)e^x$

$= -\sin x e^{\cos x} - e^x \sin(e^x)$

22) $y = \arctan(\arcsin\sqrt{x})$

$y = \tan^{-1}(\sin^{-1}\sqrt{x})$

$y' = \dfrac{\frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2\sqrt{x}}}{1 + (\sin^{-1}\sqrt{x})^2}$

$= \dfrac{1}{(2\sqrt{x}\sqrt{1-x})(1+(\sin^{-1}\sqrt{x})^2)}$

24) $xe^y = y - 1$

$xe^y \cdot y' + e^y = y'$

$y' - xe^y \cdot y' = e^y$

$y' = \dfrac{e^y}{(1 - xe^y)}$

26) $y = x^{e^x}$

$\ln y = e^x \ln x$

$\frac{1}{y} \cdot y' = e^x \cdot \frac{1}{x} + \ln x e^x$

$y' = (x^{e^x}) e^x (\frac{1}{x} + \ln x)$

y''

28) $x^6 + y^6 = 1$

$6x^5 + 6y^5 \cdot y' = 0$

$y' = \dfrac{-\cancel{6}x^5}{\cancel{6}y^5} = \boxed{\dfrac{-x^5}{y^5}}$

$y'' = \dfrac{-y^5 5x^4 + x^5 5y^4 \boxed{y'}}{y^{10}}$

$\dfrac{y^5}{y^5} = \dfrac{\left(-5x^4y^5 + 5x^5y^4\left(\frac{-x^5}{y^5}\right)\right)}{y^{10}}$

$= \dfrac{-5x^4y^{10} - 5x^{10}y^4}{y^{15}}$

$y'' = \dfrac{-5x^4y^4(y^6 + x^6)}{y^{15}}$

$= \boxed{\dfrac{-5x^4(y^6+x^6)}{y^{11}}}$