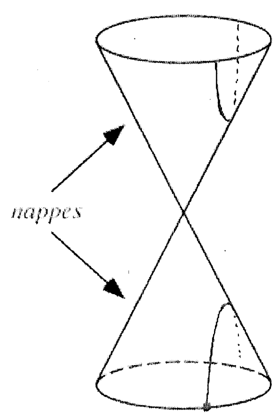
Hyperbolas

Anton 12.4

Double Right Cone



How could we slice the cone with a plane to get a hyperbola?

AT AN ∡ STEEPER THAN SLANT HT.
(NOT THRU VERTEX)

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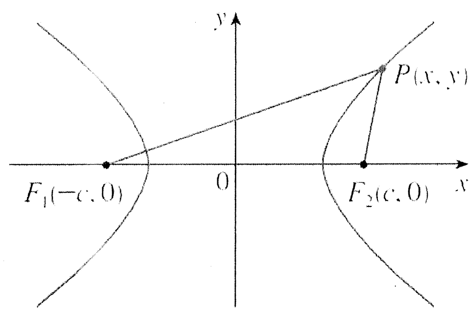
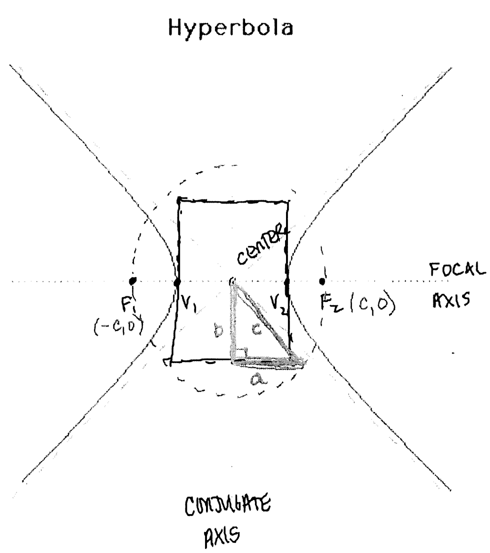
Geometric definition of a hyperbola:

A set of coplanar points the difference of whose distances from two fixes points (foci) is constant.



Some notation:



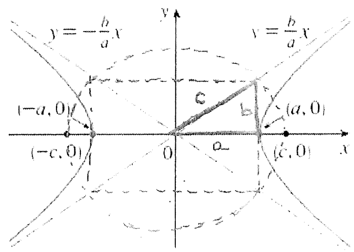
$a^2+b^2=c^2$

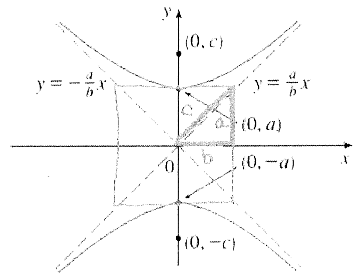
OR

$\boxed{c=\sqrt{a^2+b^2}}$

Standard Hyperbolas: C(0,0)

a is not necessarily > b





$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

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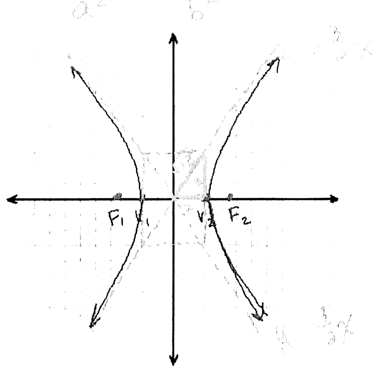
Graph the following. Identify vertices and foci.

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

$a^2 \quad b^2$

$c = \sqrt{4+9} = \sqrt{13} \Rightarrow F(\pm\sqrt{13}, 0)$

$V(\pm 2, 0)$



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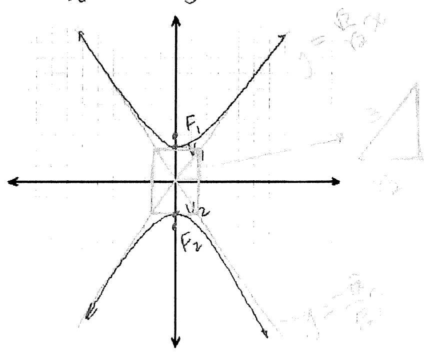
Graph the following. Identify vertices and foci.

$$\frac{y^2}{\underset{a^2}{7}} - \frac{x^2}{\underset{b^2}{2}} = 1$$

$c = \sqrt{7+2} = 3 \Rightarrow F(0, \pm 3)$

since $a = \sqrt{7} \Rightarrow V(0, \pm\sqrt{7})$



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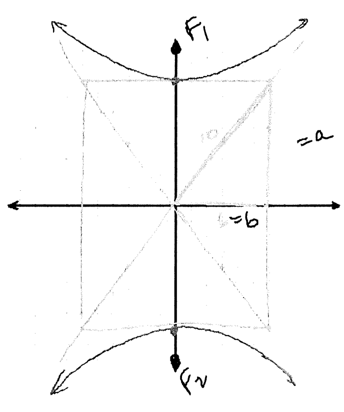
Find the equation of the hyperbola with V(0,±8) and asymptotes $y = \pm 4/3x$.



$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$\boxed{\frac{y^2}{64} - \frac{x^2}{36} = 1}$$

Foci: $c = 10$

$F(0, \pm 10)$

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Hyperbolas with Center (h,k)

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1 \longrightarrow$$ Focal axis parallel to x – axis

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1 \longrightarrow$$ Focal axis parallel to y - axis

Sketch the graph.

$$x^2-y^2-4x+8y-21=0$$

$$x^2-4x+\underline{4}-(y^2-8y+\underline{16})=21+\underline{4}+\underline{-16}$$

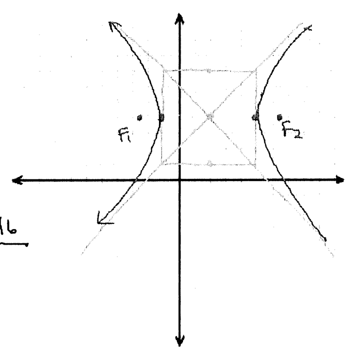
$$(x-2)^2-(y-4)^2=9$$

$$\boxed{\frac{(x-2)^2}{9}-\frac{(y-4)^2}{9}=1}$$

$a=3$

$b=3$

$\therefore c=\sqrt{3^2+3^2}=3\sqrt{2}$



$V_1(-1,4)$, $V_2(5,4)$

$F_1(2-3\sqrt{2},4)$, $F_2(2+3\sqrt{2},4)$

$\boxed{y-4=\pm 1(x-2)}$

ASYMPTOTES

Eccentricity

$$e = \frac{c}{a} \qquad \text{Recall:} \quad c = \sqrt{a^2 + b^2}$$

For the previous example: $c = 3\sqrt{2}$, $a = 3$ $\Rightarrow$ $\boxed{e = \sqrt{2}}$

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Homework: Anton 12.4

1 – 33 every other odd, 39, 41