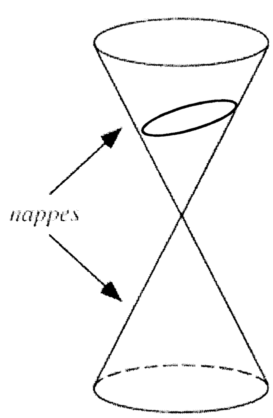
Ellipses

Anton 12.3

Double Right Cone

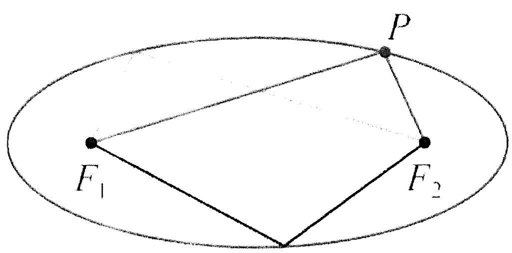


How could we slice the cone with a plane to get an ellipse?

SLICE AT AN ANGLE BETWEEN
THE SLANT HT + ⊥ TO AXIS.
(NOT THRU VERTEX)

Geometric definition of an ellipse:

A set of coplanar points the sum of whose distances from two fixes points (foci) is constant.



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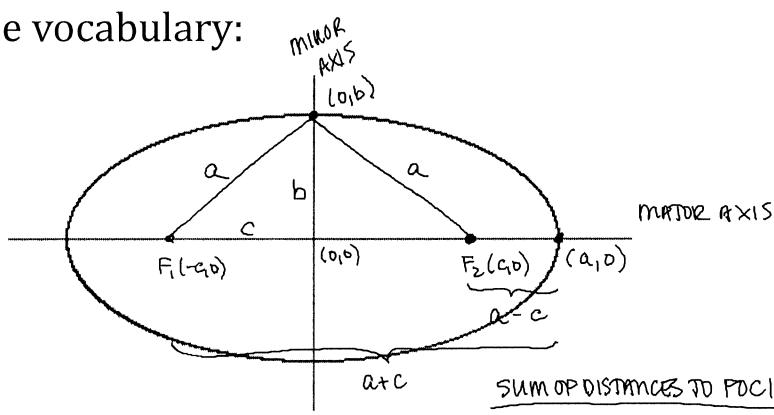
Some vocabulary:



SUM OF DISTANCES TO FOCI = 2a

$c^2+b^2=a^2$

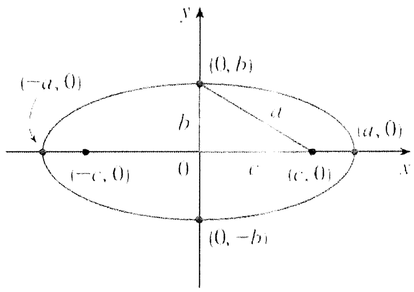
$c^2=a^2-b^2$

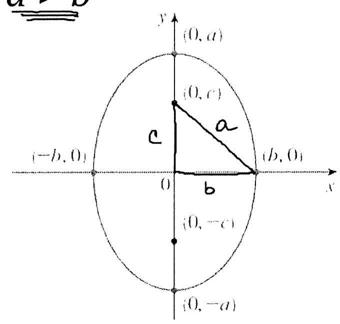
$\boxed{c=\sqrt{a^2-b^2}}$

$2a$ = length of major axis
$2b$ = length of minor axis
$2c$ = distance between foci

→

<u>Standard Ellipses</u>: C(0,0); axes of symmetry are coordinates axes; assume $a > b$





$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

→

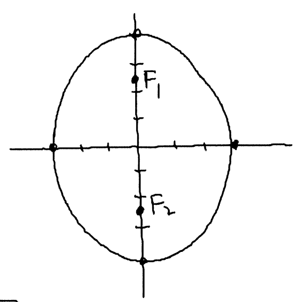
Graph the following. Identify vertices and foci.

$$\frac{16x^2}{144} + \frac{9y^2}{144} = \frac{144}{144}$$

$$\boxed{\frac{x^2}{9} + \frac{y^2}{16} = 1}$$

$$c = \sqrt{a^2 - b^2} = \sqrt{16-9} = \sqrt{7}$$

$$\boxed{F(0, \pm\sqrt{7})}$$



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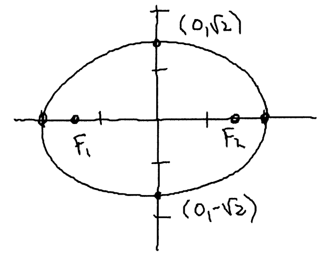
Graph the following. Identify vertices and foci.

$$\frac{x^2}{4} + \frac{2y^2}{4} = \frac{4}{4}$$

$$\boxed{\frac{x^2}{4} + \frac{y^2}{2} = 1}$$



$c = \sqrt{a^2 - b^2}$

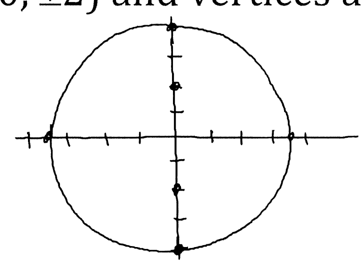
$= \sqrt{4-2}$

$c = \sqrt{2}$

$\boxed{F(\pm\sqrt{2}, 0)}$

→

Find the equation of the ellipse with foci at (0, ±2) and vertices at (0, ±4)



$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$\boxed{\frac{x^2}{12} + \frac{y^2}{16} = 1}$$

$a = 4$ $c = 2$

$c^2 = a^2 - b^2$

$4 = 16 - b^2$

$b^2 = 12$

$b = \sqrt{12}$

→

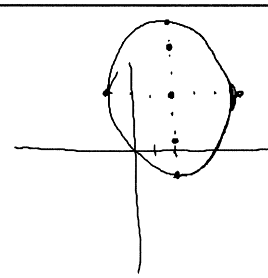
Ellipses with Center (*h*,*k*)

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 \longrightarrow$$ Major axis parallel to x – axis

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1 \longrightarrow$$ Major axis parallel to y - axis

→

Sketch the graph.



$$16x^2+9y^2-64x-54y+1=0$$

$16x^2-64x+9y^2-54y=-1$

$16(x^2-4x+\underline{4})+9(y^2-6y+\underline{9})=-1+\underline{64}+\underline{81}$

$16(x-2)^2+9(y-3)^2=144$

$\boxed{\frac{(x-2)^2}{9}+\frac{(y-3)^2}{16}=1}$

Foci: $c=\sqrt{a^2-b^2}$

$=\sqrt{16-9}$

$=\sqrt{7}$

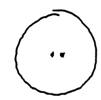
$\boxed{F(2,3\pm\sqrt{7})}$

→

Eccentricity

$$e = \frac{c}{a} \qquad \text{Recall:} \quad c = \sqrt{a^2 - b^2}$$

For the previous example: $e = \frac{\sqrt{7}}{4} \approx .66$

As e approaches 0: ELLIPSE APPROACHES A CIRCLE 

As e approaches 1: FLATTER, SKINNIER ELLIPSE 

Homework: Anton 12.3

1 – 25 every other odd, 29, 31, 33 – 39 odd