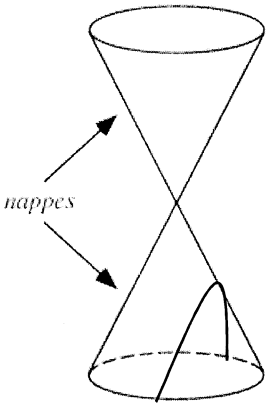
Parabolas

Anton 12.2

Double Right Cone



How could we slice the cone with a plane to get a parabola?

PARALLEL TO SLANT HT. (NOT THRU VERTEX)

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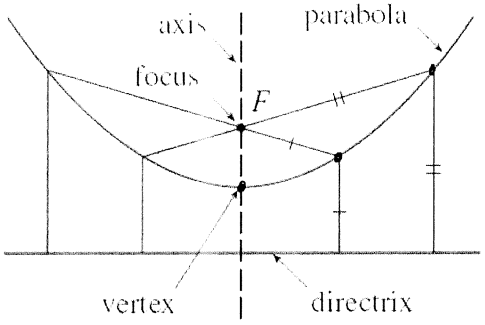
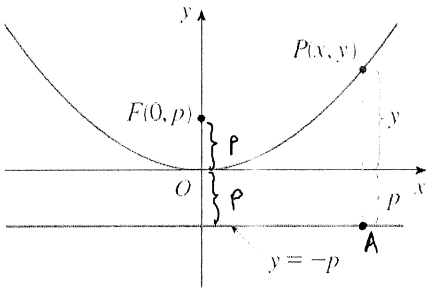
Geometric definition of a parabola:

A set of coplanar points equidistant from a point (focus) and a line (directrix.)



Find the equation of the parabola below:



$PF^2 = PA^2$

$(x-0)^2 + (y-p)^2 = (y+p)^2$

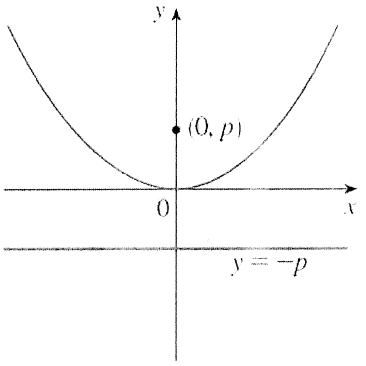
$x^2 + y^2 - 2yp + p^2 = y^2 + 2yp + p^2$

$x^2 = 4py$

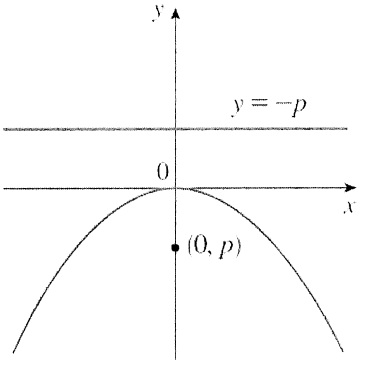
p = DISTANCE FROM FOCUS TO VERTEX

p = DISTANCE FROM VERTEX TO DIRECTRIX

Standard Parabolas: V(0,0); axis of symmetry is the y – axis.



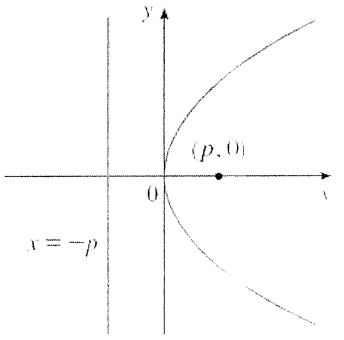
(a) $x^2 = 4py$, $p > 0$



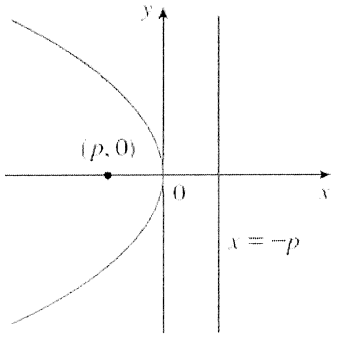
(b) $x^2 = 4py$, $p < 0$

$x^2 = -4py$ $(p > 0)$

Standard Parabolas: V(0,0); axis of symmetry is the x – axis.



(c) $y^2 = 4px$, $p > 0$



(d) $y^2 = 4px$, $p < 0$

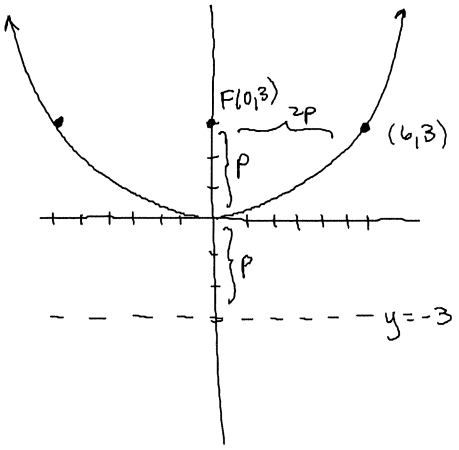
$y^2 = 4px$ $(p > 0)$

Sketch the graph.

$$x^2 = \underbrace{12}_{4p}y$$

$4p$

$p = 3$



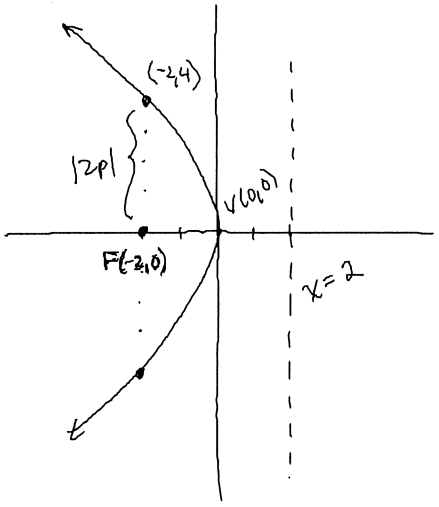
Sketch the graph.

$$y^2 + 8x = 0$$

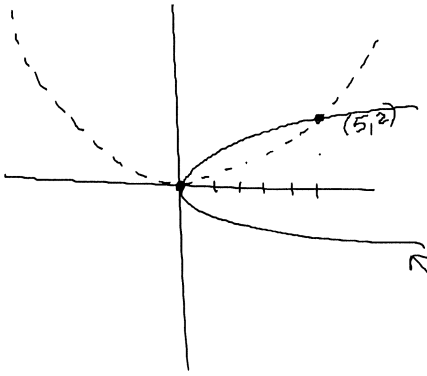
$y^2 = \underbrace{-8}_{4p}x$

$4p$

$p = -2$



Find the equation of the parabola with V(0,0) going through (5,2).



OPENS UP

$x^2 = 4py$

$5^2 = 4p(2)$

$\frac{25}{2} = 4p$

$\frac{25}{8} = p$

$x^2 = \frac{25}{2}y$

$F(0, \frac{25}{8})$

OPENS RT: $y^2 = 4px$

$2^2 = 4p(5)$

$\frac{4}{5} = 4p$

$y^2 = \frac{4}{5}x$

$F(1.5, 0)$

→

Parabolas with Vertex (h,k)

$(x-h)^2 = \pm 4p(y-k)$ ⟶ Opens up or down

$(y-k)^2 = \pm 4p(x-h)$ ⟶ Opens left or right

→

Sketch the graph.

$y^2 - 8x - 6y - 23 = 0$

$y^2 - 6y + 9 = 8x + 23 + 9$

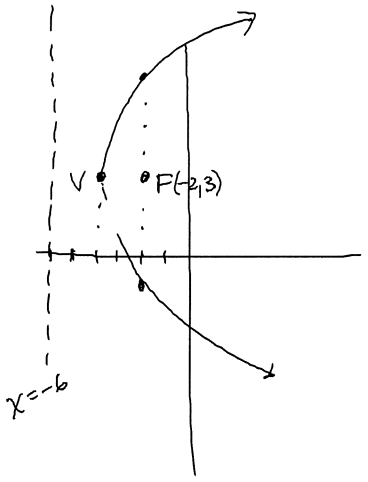
$(y-3)^2 = 8x + 32$

$\boxed{(y-3)^2 = 8(x+4)}$

$p = 2$

$V(-4, 3)$

OPENS RT



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Equations of Parabolas in Quadratic Form

$y = Ax^2 + Bx + C \quad (A \neq 0)$ Opens up or down

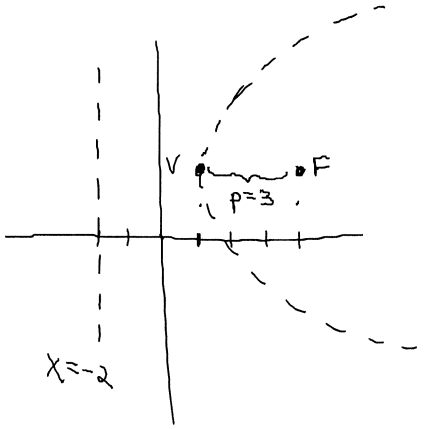
↓ $A > 0$ ↓ $A < 0$

$x = Ay^2 + By + C \quad (A \neq 0)$ Opens left or right

↓ $A < 0$ ↓ $A > 0$

→

Find the equation of the parabola with vertex (1,2) and focus (4,2).

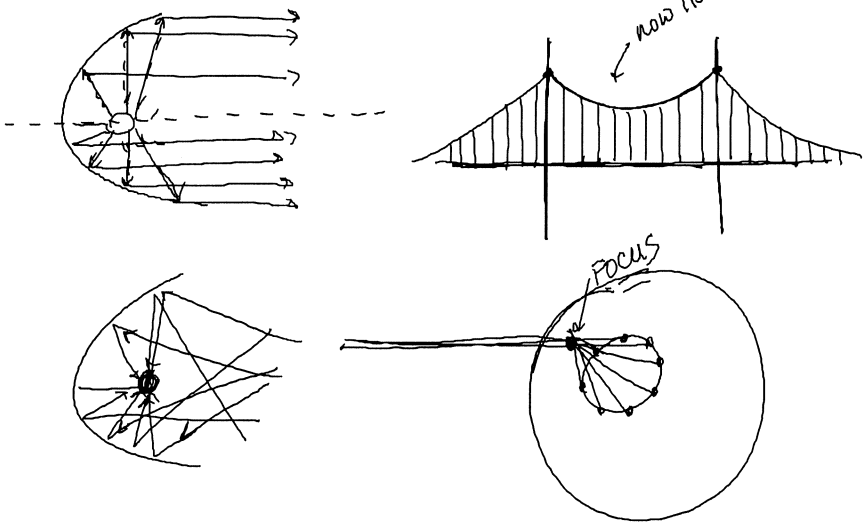


$(y-k)^2 = 4p(x-h)$

$\boxed{(y-2)^2 = 12(x-1)}$

→

What are parabolas good for?



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Homework: Anton 12.2

1 – 33 every other odd, 36, 37