Undetermined Coefficients

Anton 14.2

Agenda

1. Review HW p. 14.1 #1-23 odd
2. Determine the general solution of a nonhomogeneous second order diff. eq. using undetermined coefficients.

Second Order Nonhomogeneous Diff. Eq. with constant coefficients:

$$y'' + py' + qy = r(x)$$

Recall: The general solution of $y'' + py' + qy = 0$

is of the form: $y_c(x) = c_1 y_1(x) + c_2 y_2(x)$

where y_1 and y_2 depend on solutions to characteristic equation:

$$m^2 + pm + q = 0$$

Theorem: The general solution y_g of the second order nonhomogeneous differential equation with constant coefficients

$$y'' + py' + qy = r(x)$$

is of the form: $$y_g(x) = y_c(x) + y_p(x)$$

where y_c is the solution to the characteristic equation and y_p is any particular solution of the nonhomogeneous diff. eq.

Method of Undetermined Coefficients

Example: $y'' + 2y' - 8y = e^{3x}$

$m^2 + 2m - 8 = 0$

$(m+4)(m-2) = 0$

$m = -4, 2$

$y_c = c_1 e^{-4x} + c_2 e^{2x}$

TRY: $y_p = Ae^{3x}$

$y_p' = 3Ae^{3x}$

$y_p'' = 9Ae^{3x}$

$9Ae^{3x} + 6Ae^{3x} - 8Ae^{3x} = e^{3x}$

$7Ae^{3x} = e^{3x}$

$7A = 1$

$A = 1/7$

$\Rightarrow y_p = \frac{1}{7} e^{3x}$

$\therefore y_g = y_c + y_p$

Example: $y'' - y' - 6y = e^{3x}$

$m^2 - m - 6 = 0$

$(m-3)(m+2) = 0$

$m = 3, -2$

$$y_c = c_1 e^{3x} + c_2 e^{-2x}$$

TRY $y_p = ~~Ae^{3x}~~$ ⇒ NOT INDEPEND. OF y_c

$y_p = Axe^{3x}$

$y_p = Axe^{3x}$

$y_p' = Ax\,3e^{3x} + e^{3x} \cdot A = 3Axe^{3x} + Ae^{3x}$

$y_p'' = 9Axe^{3x} + 3Ae^{3x} + 3Ae^{3x}$

$= 9Axe^{3x} + 6Ae^{3x}$

$~~9Axe^{3x}~~ + 6Ae^{3x} - ~~3Axe^{3x}~~ - Ae^{3x} - ~~6Axe^{3x}~~ = e^{3x}$

$5Ae^{3x} = e^{3x}$

$5A = 1$

$A = 1/5$

$$y_p = \frac{1}{5}xe^{3x}$$

$$\therefore\ y_g = y_c + y_p$$

Notes:

1. If your choice for y_p is linearly dependent on y_c then you must adjust y_p .
2. Multiply by powers of x until you get what you need.

Example: $y'' + y' = 4x^2$

$m^2 + m = 0$

$m(m+1) = 0$

$m = 0, -1$

$$y_c = c_1 + c_2 e^{-x}$$

TRY ~~$y_p = A_0 + A_1 x + A_2 x^2$~~

★ THIS IS NOT INDEPENDENT OF y_c ★

$y_p = A_0 x + A_1 x^2 + A_2 x^3$

$y_p = A_0 x + A_1 x^2 + A_2 x^3$

$y_p' = A_0 + 2A_1 x + 3A_2 x^2$

$y_p'' = 2A_1 + 6A_2 x$

$2A_1 + 6A_2 x + A_0 + 2A_1 x + 3A_2 x^2 = 4x^2$

$2A_1 + A_0 = 0$

$2(-4) + A_0 = 0$

$A_0 = 8$

$6A_2 + 2A_1 = 0$

$8 + 2A_1 = 0$

$A_1 = -4$

$3A_2 = 4$

$A_2 = 4/3$

$\Rightarrow y_p = 8x + -4x^2 + 4/3 x^3$

$\therefore y_g = y_c + y_p$

Example: $y'' - 2y' + y = \sin 2x$

$m^2 - 2m + 1 = 0$

$(m-1)^2 = 0$

$y_c = c_1 e^x + c_2 x e^x$

$y_p = A_1 \sin 2x + A_2 \cos 2x$

$y_p' = 2A_1 \cos 2x - 2A_2 \sin 2x$

$y_p'' = -4A_1 \sin 2x - 4A_2 \cos 2x$

$-4A_1 \sin 2x - 4A_2 \cos 2x - 4A_1 \cos 2x + 4A_2 \sin 2x$

$+ A_1 \sin 2x + A_2 \cos 2x = \sin 2x$

$3(-3A_1 + 4A_2 = 1)$

$4(-4A_1 - 3A_2 = 0)$

$-25A_1 = 3$

$A_1 = -3/25$

$+4(+\frac{3}{25}) - 3A_2 = 0$

$-3A_2 = \frac{-12}{25}$

$A_2 = 4/25$

$y_p = -\frac{3}{25} \sin 2x + \frac{4}{25} \cos 2x$

$\therefore y_g = y_c + y_p$

If $r(x)$ is of the form...	make y_p of the form...
ke^{ax}	Ae^{ax}
$a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$	$A_0 + A_1 x + A_2 x^2 + \cdots + A_n x^n$
$a_1 \cos bx + a_2 \sin bx$	$A_1 \cos bx + A_2 \sin bx$

Homework:

Anton 14.2 # 1 – 23 odd