First Order Differential Equations

Anton 7.7

First Order Differential Equations:

Equations where the highest derivative is *dy/dx*

1. Separable: $\frac{dy}{dx} = xy$ $y(0) = -2$

$\frac{1}{y} dy = x\, dx$

$\ln|y| = \frac{1}{2}x^2 + C$

$y = Ce^{\frac{1}{2}x^2}$

$-2 = Ce^0 \Rightarrow C = -2$

$\boxed{y = -2e^{x^2/2}}$

2. First Order **Linear**

Takes the form: $$\frac{dy}{dx} + p(x)y = q(x)$$

Examples:

$$\frac{dy}{dx} + x^2 y = e^x$$ $p(x) = x^2$ $q(x) = e^x$

$$y' + y\sin x + x^3 = 0$$ $p(x) = \sin x$ $q(x) = -x^3$

$$y' = -5y + 2$$ $p(x) = 5$ $q(x) = 2$

Procedure for Solving First Order Linear Diff EQ:

1. Find the **integrating factor:** $\boxed{\mu = e^{\int p(x)dx}}$

$\mu' = e^{\int p(x)dx} \cdot p(x)$

$\mu' = \mu p(x)$

2. Multiply both sides of the diff eq. by μ:

$y' + p(x)y = q(x)$

$\mu y' + \mu p(x) y = \mu q(x)$

$\mu y' + \mu' y = \mu q(x)$

$\frac{d}{dx}(\mu y) = \mu q(x)$

3. Rewrite as: $\boxed{\frac{d}{dx}(\mu y) = \mu q(x)}$

4. Integrate both sides and solve for y

Example: $\dfrac{dy}{dx} - 4xy = x$

$$\mu = e^{\int -4x\,dx} = e^{-2x^2}$$

$$\frac{d}{dx}(\mu y) = \mu g(x)$$

$$\int \frac{d}{dx}\left(e^{-2x^2} \cdot y\right) = \int e^{-2x^2} \cdot \cancel{x}$$

$u = -2x^2$

$du = -4\cancel{x}$

$$e^{-2x^2} \cdot y = -\frac{1}{4}\int e^u\,du$$

$$e^{-2x^2} y = -\frac{1}{4}e^{-2x^2} + C$$

$$\boxed{y = -\frac{1}{4} + \frac{C}{e^{-2x^2}}}$$

$$\boxed{y = -\frac{1}{4} + Ce^{2x^2}}$$

Example: $xy' - y = x \quad y(1) = 3 \quad (x > 0)$

$$y' - \frac{1}{x}y = 1$$

$$\mu = e^{\int -1/x\,dx} = \cancel{e^{-\ln x^{-1}}} = 1/x$$

$$\frac{d}{dx}(\mu y) = \mu q(x)$$

$$\int \frac{d}{dx}\left(\frac{1}{x}\cdot y\right) = \int \frac{1}{x} \quad 1$$

$$\frac{1}{x}y = \ln x + C$$

$$y = x\ln x + xC$$

$$3 = 1\ln(1) + 1(C)$$

$$3 = C$$

$$\boxed{\begin{aligned} y &= x\ln x + 3x \\ y &= x(\ln x + 3) \end{aligned}}$$

Classwork:

Anton 7.7 # 10, 16

Homework:

Anton 7.7 # 1 – 21 odd