1.) $\int \sin t\sqrt{1+\cos t}\ dt$ $\quad u = 1+\cos t$, $du = -\sin t\,dt$

$-\int u^{1/2}\,du$

$-\frac{2}{3}u^{3/2}+C$

$\boxed{-\frac{2}{3}(1+\cos t)^{3/2}+C}$

2.) $\int \sin^2 x\cos^2 x\,dx$

$\int \frac{1}{2}(1-\cos 2x)\,\frac{1}{2}(1+\cos^2 x)\,dx$

$\frac{1}{4}\int 1-\cos^2(2x)\,dx$

$\frac{1}{4}\int 1-\frac{1}{2}(1+\cos(4x))\,dx$

$\frac{1}{4}\left[x-\frac{1}{2}\left(x+\frac{1}{4}\sin(4x)\right)\right]+C$

$\frac{1}{4}\left[\cancel{x}-\frac{1}{2}x-\frac{1}{8}\sin(4x)\right]+C$ $\quad \frac{1}{2}x$

$\boxed{\frac{1}{8}x-\frac{1}{32}\sin(4x)+C}$

3.) $\int \cos^3 x\,dx$

$\int \cos^2 x\cdot\cos x\,dx$ $\quad u=\sin x$, $du=\cos x\,dx$

$\int(1-\sin^2 x)\cos x\,dx$

$\int 1-u^2\,du$

$u-\frac{1}{3}u^3+C$

$\boxed{\sin x-\frac{1}{3}\sin^3 x+C}$

4.) $\int \sin^2(2t)\,dt$

$\frac{1}{2}\int 1-\cos(4t)\,dt$

$\frac{1}{2}\left(t-\frac{1}{4}\sin(4t)\right)+C$

$\boxed{\frac{1}{2}t-\frac{1}{8}\sin(4t)+C}$

5.) $\int \sin^3 x\cos^2 x\,dx$ $\quad u=\cos x$, $du=-\sin x\,dx$

$\int \sin^2 x\cos^2 x\cdot\sin x\,dx$

$\int(1-\cos^2 x)\cos^2 x\,\sin x\,dx$

$-\int(1-u^2)u^2\,du$

$-\int u^2-u^4\,du$

$-\frac{1}{3}u^3+\frac{1}{5}u^5+C$

$\boxed{-\frac{1}{3}\cos^3 x+\frac{1}{5}\cos^5 x+C}$

6.) $\int \cos^2(3\theta)\,d\theta$

$\frac{1}{2}\int 1+\cos(6\theta)\,d\theta$

$\frac{1}{2}\left[\theta+\frac{1}{6}\sin(6\theta)\right]+C$

$\boxed{\frac{1}{2}\theta+\frac{1}{12}\sin(6\theta)+C}$

7) $\int \sec^4(3x)\tan(3x)\,dx$

$\int \sec^3(3x)\sec(3x)\tan(3x)\,dx$ $\quad u=\sec(3x)$, $du=\frac{1}{3}\sec(3x)\tan(3x)\,dx$

$\frac{1}{3}\int u^3\,du$

$\frac{1}{3}\cdot\frac{1}{4}u^4+C$

$\boxed{\frac{1}{12}\sec^4(3x)+C}$

-OR- $\int \sec^4(3x)\tan(3x)\,dx$ $\quad u=\tan(3x)$, $du=3\sec^2(3x)\,dx$

$\int \sec^2(3x)\sec^2(3x)\tan(3x)\,dx$

$\int (1+\tan^2(3x))\tan(3x)\sec^2(3x)\,dx$

$\frac{1}{3}\int (1+u^2)u\,du$

$\frac{1}{3}\int u+u^3\,du$

$\frac{1}{3}\left[\frac{1}{2}u^2+\frac{1}{4}u^4\right]+C$

$\boxed{\frac{1}{6}\tan^2(3x)+\frac{1}{12}\tan^4(3x)+C}$

8) $\int \sin^4(ax)\,dx$

$\int \sin^2(ax)\sin^2(ax)\,dx$

$\int \frac{1}{2}(1-\cos(2ax))\cdot\frac{1}{2}(1-\cos(2ax))\,dx$

$\frac{1}{4}\left[\int 1-2\cos(2ax)+\int \cos^2(2ax)\,dx\right]$

$\frac{1}{2}\int 1+\cos(4ax)$

so, $\frac{1}{4}\left[x-\frac{1}{2a}\sin(2ax)+\frac{1}{2}\left(x+\frac{1}{4a}\sin(4ax)\right)\right]$

$\frac{1}{2}x+\frac{1}{8a}\sin(4ax)$

$\frac{1}{4}x-\frac{1}{4a}\sin(2ax)+\frac{1}{8}x+\frac{1}{32a}\sin(4ax)$

$\boxed{\frac{3}{8}x-\frac{1}{4a}\sin(2ax)+\frac{1}{32a}\sin(4ax)+C}$

9) $\int \frac{\cos^3 t}{\sin^2 t}\,dt$

$\int \frac{\cos^2 t}{\sin^2 t}\cdot\cos t\,dt$ $\quad u=\sin t$, $du=\cos t\,dt$

$\int \frac{(1-\sin^2 t)}{\sin^2 t}\cos t\,dt$

$\int \frac{1-u^2}{u^2}\,du$

$\int \frac{1}{u^2}-1\,du$

u^{-2}

$-1u^{-1}-u+C$

$\frac{-1}{\sin t}-\sin t+C$

$\boxed{-\csc t-\sin t+C}$