

CALCULUS 2

Name: KE-1

CHAPTER 9 PRACTICE TEST

1. $\int x\sqrt{3x-1} dx$ (rationalize)

2. $\int \frac{x^2-3}{x+4} dx$ (long division)

3. $\int x \cos 3x dx$ (parts)

4. $\int x^2 \ln x dx$ (parts)

5. $\int 2x^2 e^{-x} dx$ (tabular)

6. $\int x^3 \sin 2x dx$ (tabular)

7. $\int \frac{4x-1}{x^2-4} dx$ (partial fractions)

8. $\int \frac{x+1}{(x-1)^2} dx$ (partial fractions)

Use any appropriate method:

9. $\int \ln(x+2) dx$

10. $\int \sin^{-1} 2x dx$

11. $\int \frac{1}{1+9x^2} dx$

12. $\int \frac{5x}{1-25x^2} dx$

13. $\int \frac{x^4-3x^2+2}{x^2-4} dx$

14. $\int x\sqrt{4x^2-1} dx$

15. $\int \frac{1}{1-x^2} dx$

16. $\int \frac{x-9}{x^3-6x^2+9x} dx$

CALC 2

CH 9 PT KEY

1. $\int x\sqrt{3x-1} dx$ $u = \sqrt{3x-1}$
 $u^2 = 3x-1$
 $x = \frac{1}{3}u^2 + \frac{1}{3}$
 $dx = \frac{2}{3}u du$
 $= \int \left(\frac{1}{3}u^2 + \frac{1}{3}\right)u \cdot \frac{2}{3}u du$
 $= \int \frac{2}{9}u^4 + \frac{2}{9}u^2 du$
 $= \frac{2}{45}u^5 + \frac{2}{27}u^3 + c$
 $= \frac{2}{45}\sqrt{3x-1}^5 + \frac{2}{27}\sqrt{3x-1}^3 + c$

2. $\int \frac{x^2-3}{x+4} dx$ $\begin{array}{r} x-4 \\ x+4 \overline{) x^2+0x-3} \\ \underline{x^2+4x} \\ -4x-3 \\ \underline{-4x-16} \\ 13 \end{array}$
 $= \int x-4 + \frac{13}{x+4} dx$
 $= \frac{1}{2}x^2 - 4x + 13 \ln|x+4| + c$

3. $\int x \cos 3x dx$ $u = x$ $dv = \cos 3x dx$
 $du = 1 dx$ $v = \frac{1}{3} \sin 3x$
 $\frac{1}{3} x \sin 3x - \int \frac{1}{3} \sin 3x dx$
 $\frac{1}{3} x \sin 3x + \frac{1}{9} \cos 3x + c$

4. $\int x^2 \ln x dx$ $u = \ln x$ $dv = x^2 dx$
 $du = \frac{1}{x} dx$ $v = \frac{1}{3} x^3$
 $\frac{1}{3} x^3 \ln x - \int \frac{1}{3} x^2 dx$
 $\frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 + c$

5. $\int 2x^2 e^{-x} dx$

u	dv	
$+ 2x^2$	$e^{-x} dx$	$= -2x^2 e^{-x} - 4x e^{-x} - 4e^{-x} + c$
$- 4x$	$-e^{-x}$	
$+ 4$	e^{-x}	
0	$-e^{-x}$	

6. $\int x^3 \sin 2x \, dx$

	u	dv	
	$+ x^3$	$\sin 2x \, dx$	$= -\frac{1}{2}x^3 \cos 2x + \frac{3}{4}x^2 \sin 2x + \frac{3}{4}x \cos 2x - \frac{3}{8} \sin 2x + c$
	$- 3x^2$	$-\frac{1}{2} \cos 2x$	
	$+ 6x$	$-\frac{1}{4} \sin 2x$	
	$- 6$	$\frac{1}{8} \cos 2x$	
	0	$-\frac{1}{16} \sin 2x$	

7. $\int \frac{4x-1}{x^2-4} \, dx = \int \frac{4x-1}{(x+2)(x-2)} \, dx = \int \frac{A}{x+2} + \frac{B}{x-2} \, dx$

$$4x-1 = A(x-2) + B(x+2)$$

$$x=2: 7 = 4B \quad B = \frac{7}{4}$$

$$x=-2: -9 = -4A \quad A = \frac{9}{4}$$

$$= \int \frac{9}{4(x+2)} + \frac{7}{4(x-2)} \, dx$$

$$= \frac{9}{4} \ln|x+2| + \frac{7}{4} \ln|x-2| + c$$

8. $\int \frac{x+1}{(x-1)^2} \, dx = \int \frac{A}{x-1} + \frac{B}{(x-1)^2} \, dx$

$$x+1 = A(x-1) + B$$

$$x=1: 2 = B$$

$$x=0: 1 = -A + B \rightarrow 1 = -A + 2 \quad A = 1$$

$$\int \frac{1}{x-1} + \frac{2}{(x-1)^2} \, dx$$

$$= \ln|x-1| - \frac{2}{x-1} + c$$

$$9. \int \ln(x+2) dx \quad u = \ln(x+2) \quad du = dx$$

$$du = \frac{1}{x+2} dx \quad v = x$$

$$= x \ln(x+2) - \int \frac{x}{x+2} dx$$

$$\frac{1}{x+2} \int \frac{x+0}{x+2}$$

$$= x \ln(x+2) - \int 1 - \frac{2}{x+2} dx$$

$$\frac{x+2}{x+2}$$

$$-2$$

$$= x \ln(x+2) - x + 2 \ln|x+2| + C$$

$$10. \int \sin^{-1} 2x dx \quad u = \sin^{-1} 2x \quad du = dx$$

$$du = \frac{2}{\sqrt{1-4x^2}} dx \quad v = x$$

$$= x \sin^{-1} 2x - \int \frac{2x}{\sqrt{1-4x^2}} dx \quad u = 1-4x^2$$

$$du = -8x \quad -\frac{1}{4} du = 2x$$

$$- \int -\frac{1}{4} \cdot \frac{1}{u} du$$

$$= x \sin^{-1} 2x + \frac{1}{2} \sqrt{1-4x^2} + C$$

$$11. \int \frac{1}{1+9x^2} dx = \frac{1}{3} \tan^{-1} 3x + C$$

$$a=1 \quad u=3x$$

$$du = 3 dx$$

$$\frac{1}{3} du = dx$$

$$\int \frac{1}{1+u^2} du$$

$$\frac{1}{3} \tan^{-1} 3x + C$$

$$12. \int \frac{-5x}{1-25x^2} dx \quad u = 1-25x^2$$

$$du = -50x dx \quad -\frac{1}{10} du = 5x dx$$

$$= \int -\frac{1}{10} \cdot \frac{1}{u} du$$

$$= -\frac{1}{10} \ln|1-25x^2| + C$$

$$13. \int \frac{x^4 - 3x^2 + 2}{x^2 - 4} dx$$

$$= \int \frac{x^2 + 1}{x^2 - 4} + \frac{6}{x^2 - 4} dx$$

$$\frac{6}{x^2 - 4} = \frac{A}{x+2} + \frac{B}{x-2}$$

$$6 = A(x-2) + B(x+2)$$

$$x=2: 6 = 4B \quad B = \frac{3}{2}$$

$$x=-2: 6 = -4A \quad A = -\frac{3}{2}$$

$$= \int x^2 + 1 + \frac{-\frac{3}{2}}{x+2} + \frac{\frac{3}{2}}{x-2} dx$$

$$= \frac{1}{3}x^3 + x - \frac{3}{2} \ln|x+2| + \frac{3}{2} \ln|x-2| + c$$

$$14. \int x \sqrt{4x^2 - 1} dx \quad u = 4x^2 - 1$$

$$du = 8x dx \quad \frac{1}{8} du = x dx$$

$$= \int \frac{1}{8} \sqrt{u} du$$

$$= \frac{1}{12} \sqrt{(4x^2 - 1)^3} + c$$

$$15. \int \frac{1}{1-x^2} dx = \frac{A}{1-x} + \frac{B}{1+x}$$

$$1 = A(1+x) + B(1-x)$$

$$x=-1: 1 = 2B \quad B = \frac{1}{2}$$

$$x=1: 1 = 2A \quad A = \frac{1}{2}$$

$$\int \frac{\frac{1}{2}}{1-x} + \frac{\frac{1}{2}}{1+x} dx$$

$$= -\frac{1}{2} \ln|1-x| + \frac{1}{2} \ln|1+x| + c$$

16.

$$\int \frac{x-9}{x^3-6x^2+9x} dx = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{(x-3)^2}$$

$$x-9 = A(x-3)^2 + B(x-3)x + Cx$$

$$x=3: -6 = 3C \quad C=-2$$

$$x=0: -9 = 9A \quad A=-1$$

$$x=1: -8 = 4A - 2B + C$$

$$-8 = -4 - 2B - 2$$

$$-2 = -2B \quad B=1$$

$$= \int \frac{-1}{x} + \frac{1}{x+3} + \frac{-2}{(x-3)^2}$$

$$= -\ln|x| + \ln|x+3| + \frac{2}{x-3} + C$$