Calculus 2 Name Key

Worksheet - Growth and Decay Date

1) The population of a particular city doubled in 60 years from 1890 to 1950. If the rate of a natural increase of the population at any time is proportional to the population at that time, and the population in 1950 was 60,000, estimate the population in the year 2000.

$2 = 1e^{K(60)}$

$\ln 2 = 60K$

$K \approx .01155$

$y = 60{,}000e^{K(50)}$

$\boxed{y = 106{,}894}$

2) In a certain bacterial culture where the rate of growth of bacteria is proportional to the number present, the number triples in one hour. If at the end of 4 hours there were 10 million bacteria, how many bacteria were present initially?

$3 = 1e^{k(1)}$

$\ln 3 = K$

$K \approx 1.098612$

$10 = Y_0 e^{K(4)}$

$Y_0 = 10/e^{K(4)}$

$\boxed{Y_0 = .1234569 \text{ million} \approx 123{,}456.9}$

3) Thirty percent of a radioactive substance disappears in 15 years. Find the half-life of the substance.

70 % left.

$70 = 100e^{K(15)}$,

$K \approx -.023778$

$50 = 100e^{Kt}$

$\frac{\ln .5}{K} = \frac{Kt}{K}$

$\boxed{t = 29.15 \text{ yrs.}}$

4) A bacteria culture starts with 500 bacteria and after 3 hours there are 8000 bacteria.
a) Find an expression for the number of bacteria *t* hours.
b) Find the number of bacteria after 4 hours.
c) When will the population reach 30,000?

a.) $8000 = 500e^{K(3)}$

$K = .924196$

$\boxed{y = 500e^{.924196t}}$

b.) $y = 500e^{K(4)}$

$\boxed{y = 20{,}158.7}$

c.) $30{,}000 = 500e^{Kt}$

$\frac{\ln 60}{K} = \frac{Kt}{K}$

$\boxed{t = 4.43 \text{ hrs.}}$

5) A common inhabitant of human intestines is the bacterium *Escherichia coli*. A cell of this bacterium in a nutrient broth medium divides into two cells every 20 minutes. The initial population of a culture is 100 cells.

$= 1/3$ hrs.

a) Find an expression for the number of cells after t hours.
b) Find the number of cells after 10 hours.
c) When will the population reach 10,000 cells?

a.) $2 = 1e^{K(1/3)}$

$\ln 2 = \frac{1}{3}K$

$K = 2.07944$

$\boxed{y = 100e^{2.07944t}}$

b.) $y = 100e^{0.07944(10)}$

$\boxed{y = 1.074 \times 10^{11}}$

c.) $10{,}000 = 100e^{Kt}$

$\frac{\ln 100}{K} = \frac{Kt}{K}$

$\boxed{t = 2.21 \text{ hrs.}}$

6) Find the age of an ancient skull that is measured to have 40% of the ^{14}C it contained while living. The half-life of ^{14}C is 5,730 years.

$1 = 2e^{K(5,730)}$

$K \approx -.000121$

$40 = 100e^{K \cdot t}$

$\frac{\ln .4}{K} = \frac{Kt}{K}$

$\boxed{t = 7572.65 \text{ yrs.}}$

7) The rate of natural increase of a certain city is proportional to the population. If the population increases from 40,000 to 60,000 in 40 years, when will the population be 80,000?

$60{,}000 = 40{,}000e^{K(40)}$

$K \approx .0101366$

$80{,}000 = 40{,}000e^{K(t)}$

$\ln 2 = Kt$

$\boxed{t = 68.38 \text{ yrs.}}$

8) If the half-life of a certain material is 57 years, how long will it take for the material become 25% of its original size?

$1 = 2e^{K(57)}$

$K = -.0121605$

$25 = 100e^{Kt}$

$\ln .25 = Kt$

$\boxed{t = 114 \text{ yrs.}}$