CHAPTER 9 TEST #1 REVIEW EXAMPLES

DETERMINE IF THE SERIES CONVERGES ABSOLUTELY, CONVERGES CONDITIONALLY, OR DIVERGES.

① $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ ② $\sum_{n=1}^{\infty} \frac{n+1}{n!}$ ③ $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ ④ $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$

⑤ $\sum \frac{(-1)^n (n^2+1)}{2n^2+n-1}$ ⑥ $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2-1}}$ ⑦ $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^n$

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① $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} = \frac{-1}{\sqrt{1}} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} \cdots$

THE ABSOLUTE VALUES OF TERMS ARE DECREASING AND APPROACHING ZERO $\Rightarrow$ THE SERIES CONV. BY ALT. SERIES TEST.

$\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{\sqrt{n}} \right| = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ DIVERGES (P-SERIES, $p=2$).

$\therefore \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ CONVERGES CONDITIONALLY

② RATIO TEST: $\rho = \lim_{n\to\infty} \left| \frac{n+2}{(n+1)\cancel{n!}} \cdot \frac{\cancel{n!}}{n+1} \right| = \frac{1}{n+1} \to 0 < 1$

$\therefore \sum_{n=1}^{\infty} \frac{n+1}{n!}$ CONVERGES ABSOLUTELY

③ INTEGRAL TEST: $\int_2^{\infty} \frac{1}{x(\ln x)^2}\,dx$ $\quad u = \ln x$, $du = 1/x$

$= \int_{\ln 2}^{\infty} \frac{1}{u^2}\,du$

$= \left. -\frac{1}{u} \right]_{\ln 2}^{\infty} = \frac{-1}{\infty} + \frac{1}{\ln 2} \to$ CONVERGES

$\therefore \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ CONVERGES ABSOLUTELY

④ RATIO TEST: $\rho = \lim_{n\to\infty} \left| \frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n} \right| = \left| \frac{3}{n+1} \right| \to 0 < 1$

$\therefore \sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$ CONVERGES ABSOLUTELY

⑤ DIVERGENCE TEST: $\lim_{n\to\infty} \frac{(-1)^n (n^2+1)}{2n^2+n-1}$ DOES NOT EXIST

$\therefore \sum \frac{(-1)^n (n^2+1)}{2n^2+n-1}$ DIVERGES

⑥ BEHAVES LIKE A P-SERIES W/ $p=2 \Rightarrow$ CONVERGES ABSOLUTELY

⑦ DIVERGENCE TEST: $\lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n$

$= \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^{-n}$

* SEE BELOW

$= \lim_{n\to\infty} (1+1/n)^{-n} = 1/e \neq 0$

$\therefore \sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^n$ DIVERGES

$\lim_{n\to\infty} (1+1/n)^{-n} \qquad (1)^{-\infty}$

$y = (1+1/n)^{-n}$

$\ln y = -n\ln(1+1/n)$

$= \frac{\ln(1+1/n)}{-1/n} \quad \left(\frac{0}{0}\right)$

$\stackrel{L}{=} \frac{1/(1+1/n) \cdot -1/n^2}{1/n^2} = \frac{-1}{1+1/n} \to -1$

$\ln y \to -1$

$y \to e^{-1}$