AP Calculus BC
Chapter 9 AP Exam Problems

All problems are NON CALCULATOR unless otherwise indicated.

1. If $S_n = \left(\dfrac{(5+n)^{100}}{5^{n+1}}\right)\left(\dfrac{5^n}{(4+n)^{100}}\right)$, to what number does the sequence $\{S_n\}$ converge?

A) $\dfrac{1}{5}$ B) 1 C) $\dfrac{3}{4}$ D) $\left(\dfrac{5}{4}\right)^{100}$ E) Does not converge

2. Which of the following series are convergent?

I. $1+\dfrac{1}{2^2}+\dfrac{1}{3^2}+\cdots+\dfrac{1}{n^2}+\cdots$

II. $1+\dfrac{1}{2}+\dfrac{1}{3}+\cdots+\dfrac{1}{n}+\cdots$

III. $1-\dfrac{1}{3}+\dfrac{1}{3^2}-\cdots+\dfrac{(-1)^{n+1}}{3^{n-1}}+\cdots$

A) I only
B) III only
C) I and III
D) II and III
E) I, II, and III

3. Which of the following series converge?

I. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n+1}$ II. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{3}{2}\right)^n$ III. $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\ln n}$

A) I only
B) II only
C) III only
D) I and III
E) I, II, and III

4. Which of the following series diverge?

I. $\displaystyle\sum_{k=3}^{\infty}\frac{1}{k^2+1}$ II. $\displaystyle\sum_{k=1}^{\infty}\left(\frac{6}{7}\right)^k$ III. $\displaystyle\sum_{k=2}^{\infty}\frac{(-1)^k}{k}$

A) None
B) II only
C) III only
D) I and III
E) II and III

AP Calculus BC
Chapter 9 AP Exam Problems

5. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n}{n+2}$ II. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$ III. $\sum_{n=1}^{\infty} \frac{1}{n}$

A) None
B) II only
C) III only
D) I and II
E) I and III

6. If $\lim_{b\to\infty} \int_1^b \frac{dx}{x^p}$ is finite, which of the following must be true?

A) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges
B) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges
C) $\sum_{n=1}^{\infty} \frac{1}{n^{p-2}}$ converges
D) $\sum_{n=1}^{\infty} \frac{1}{n^{p-1}}$ converges
E) $\sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$ diverges

7. For what integer k, $k > 1$, will both $\sum_{n=2}^{\infty} \frac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\frac{k}{4}\right)^n$ converge?

A) 6 B) 5 C) 4 D) 3 E) 2

8. For $-1 < x < 1$ if $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n-1}}{2n-1}$, then $f'(x) =$

A) $\sum_{n=1}^{\infty} (-1)^{n+1} x^{2n-2}$
B) $\sum_{n=1}^{\infty} (-1)^{n} x^{2n-2}$
C) $\sum_{n=1}^{\infty} (-1)^{2n} x^{2n}$
D) $\sum_{n=1}^{\infty} (-1)^{n} x^{2n}$
E) $\sum_{n=1}^{\infty} (-1)^{n+1} x^{2n}$

9. The coefficient for x^3 in the Taylor series for e^{3x} about $x = 0$ is

A) $\frac{1}{6}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{3}{2}$ E) $\frac{9}{2}$

AP Calculus BC
Chapter 9 AP Exam Problems

10. Which of the following is a series expansion of $\sin(2x)$?

A) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots+\frac{(-1)^{n-1}x^{2n-1}}{(2n-1)!}+\cdots$

B) $2x-\frac{(2x)^3}{3!}+\frac{(2x)^5}{5!}-\cdots+\frac{(-1)^{n-1}(2x)^{2n-1}}{(2n-1)!}+\cdots$

C) $-\frac{(2x)^2}{2!}+\frac{(2x)^4}{4!}-\cdots+\frac{(-1)^n(2x)^{2n}}{(2n)!}+\cdots$

D) $\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots+\frac{x^{2n}}{(2n)!}+\cdots$

E) $2x+\frac{(2x)^3}{3!}+\frac{(2x)^5}{5!}+\cdots+\frac{(2x)^{2n-1}}{(2n-1)!}+\cdots$

11. The coefficient of x^6 in the Taylor series expansion about $x = 0$ for $f(x)=\sin\left(x^2\right)$ is

A) $-\frac{1}{6}$ B) 0 C) $\frac{1}{120}$ D) $\frac{1}{6}$ E) 1

12. What is the approximation of the value of $\sin 1$ obtained by using the fifth-degree Taylor polynomial about $x = 0$ for $\sin x$?

A) $1-\frac{1}{2}+\frac{1}{24}$

B) $1-\frac{1}{2}+\frac{1}{4}$

C) $1-\frac{1}{3}+\frac{1}{5}$

D) $1-\frac{1}{4}+\frac{1}{8}$

E) $1-\frac{1}{6}+\frac{1}{120}$

13. If $\sum_{n=0}^{\infty} a_n x^n$ is a Taylor series that converges to $f(x)$ for all real x, then $f'(1)=$

A) 0

B) a_1

C) $\sum_{n=0}^{\infty} a_n$

D) $\sum_{n=1}^{\infty} na_n$

E) $\sum_{n=1}^{\infty} na_n x^{n-1}$

14. (**CALCULATOR PROBLEM**) The graph of the function represented by the Maclaurin series $1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\cdots+\frac{(-1)^n x^n}{n!}+\cdots$ intersects the graph of $y=x^3$ at $x =$

A) 0.773 B) 0.865 C) 0.929 D) 1.000 E) 1.857

15. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$ converges?

A) $-1 \le x < 1$ C) $0 < x < 2$ E) $0 \le x \le 2$
B) $-1 \le x \le 1$ D) $0 \le x < 2$

16. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{x^n}{n}$ converges?

A) $-1 \le x \le 1$ C) $-1 \le x < 1$ E) all real x
B) $-1 < x \le 1$ D) $-1 < x < 1$

17. The interval of convergence of $\sum_{n=0}^{\infty} \frac{(x-1)^n}{3^n}$ is

A) $-3 < x \le 3$ C) $-2 < x < 4$ E) $0 \le x \le 2$
B) $-3 \le x \le 3$ D) $-2 \le x < 4$

18. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}}$ converges?

A) $-3 < x < -1$ C) $-3 \le x \le -1$ E) $-1 \le x \le 1$
B) $-3 \le x < -1$ D) $-1 \le x < 1$

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19. (1990 BC5) Let f be the function defined by $f(x) = \frac{1}{x-1}$.

(a) Write the first four terms and the general term of the Taylor series expansion of $f(x)$ about $x = 2$.

(b) Use the result from part (a) to find the first four terms and the general term of the series expansion about $x = 2$ for $\ln|x-1|$.

(c) Use the series in part (b) to compute a number that differs from $\ln\frac{3}{2}$ by less than 0.05. Justify your answer.

AP Calculus BC
Chapter 9 AP Exam Problems

20. (1992 BC6) Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^p \ln n}$, where $p>0$.

(a) Show that the series converges for $p > 1$.
(b) Determine whether the series converges or diverges for $p = 1$. Show your analysis.
(c) Show that the series diverges for $0 \le p < 1$.

21. (1995 BC4) Let f be a function that has derivatives of all orders for all real numbers. Assume $f(1)=3,\ f'(1)=-2,\ f''(1)=2,\ \textit{and}\ f'''(1)=4.$

(a) Write the second-degree Taylor polynomial for f about $x = 1$ and use it to approximate $f(0.7)$.
(b) Write the third-degree Taylor polynomial for f about $x = 1$ and use it to approximate $f(1.2)$.
(c) Write the second-degree Taylor polynomial for f', the derivative of f, about $x = 1$ and use it to approximate $f'(1.2)$.

22. (1997 BC2) Let $P(x)=7-3(x-4)+5(x-4)^2-2(x-4)^3+6(x-4)^4$ be the fourth-degree Taylor polynomial for the function f about $x = 4$. Assume f has derivatives of all orders for all real numbers.

(a) Find $f(4)$ and $f'''(4)$.
(b) Write the second-degree Taylor polynomial for f' about $x = 4$ and use it to approximate $f'(4.3)$.
(c) Write the fourth-degree Taylor polynomial for $g(x)=\int_4^x f(t)dt$ about 4.
(d) Can $f(3)$ be determined from the information given? Justify your answer.

23. (1998 BC3) Let f be a function that has derivatives of all orders for all real numbers. Assume $f(0)=5,\ f'(0)=-3,\ f''(0)=1,\ \textit{and}\ f'''(0)=4.$

(a) Write the third-degree Taylor polynomial for f about $x = 0$ and use it to approximate $f(0.2)$.
(b) Write the fourth-degree Taylor polynomial for g, where $g(x)=f\left(x^2\right)$, about $x = 0$.
(c) Write the third-degree Taylor polynomial for h, where $h(x)=\int_0^x f(t)dt$, about $x = 0$.
(d) Let h be defined as in part (c). Given that $f(1)=3$, either find the exact value of $h(1)$ or explain why it cannot be determined.

AP Calculus BC
Chapter 9 AP Exam Problems

24. (1999 BC4) The function *f* has derivatives of all orders for all real numbers *x*. Assume that $f(2) = -3,\ f'(2) = 5,\ f''(2) = 3,\ and\ f'''(2) = -8$.

(a) Write the third-degree Taylor polynomial for *f* about *x* = 2 and use it to approximate $f(1.5)$.

(b) The fourth derivative of *f* satisfies the inequality $\left|f^4(x)\right| \le 3$ for all *x* in the closed interval [1.5, 2]. Use the Lagrange error bound on the approximation to $f(1.5)$ found in part (a) to explain why $f(1.5) \ne -5$.

(c) Write the fourth-degree Taylor polynomial, $P(x)$, for $g(x) = f\left(x^2 + 2\right)$ about *x* = 0. Use *P* to explain why *g* must have a relative minimum at *x* = 0.

25. (2001 BC6) A function *f* is defined by $f(x) = \frac{1}{3} + \frac{2}{3^2}x + \frac{3}{3^3}x^2 + \cdots + \frac{n+1}{3^{n+1}}x^n + \cdots$ for all *x* in the interval of convergence of the given power series.

(a) Find the interval of convergence for this power series. Show the work that leads to your answer.

(b) Find $\lim\limits_{x \to 0} \frac{f(x) - \frac{1}{3}}{x}$.

(c) Write the first three nonzero terms and the general term for an infinite series that represents $\int_0^1 f(x)dx$.

(d) Find the sum of the series determined in part (c).

26. (2002 BC6) The Maclaurin series for the function *f* is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{(2x)^{n+1}}{n+1} = 2x + \frac{4x^2}{2} + \frac{8x^3}{3} + \frac{16x^4}{4} + \cdots + \frac{(2x)^{n+1}}{n+1} + \cdots$$

(a) Find the interval of convergence of the Maclaurin series for *f*. Justify your answer.

(b) Find the first four terms and the general term of the Maclaurin series for $f'(x)$.

(c) Use the Maclaurin series you found in part (b) to find the value of $f'\left(-\frac{1}{3}\right)$.

AP Calculus BC
Chapter 9 AP Exam Problems

27. (2002B BC6) The Maclaurin series for $\ln\left(\frac{1}{1-x}\right)$ is $\sum_{n=1}^{\infty}\frac{x^n}{n}$ with interval of convergence $-1 \le x < 1$.

(a) Find the Maclaurin series for $\ln\left(\frac{1}{1+3x}\right)$ and determine the interval of convergence.

(b) Find the value of $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$.

(c) Give a value of p such that $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^p}$ converges, but $\sum_{n=1}^{\infty}\frac{1}{n^{2p}}$ diverges. Give reasons why your value of p is correct.

(d) Give a value of p such that $\sum_{n=1}^{\infty}\frac{1}{n^p}$ diverges, but $\sum_{n=1}^{\infty}\frac{1}{n^{2p}}$ converges. Give reasons why your value of p is correct.

28. (2003 BC6) The function f is defined by the power series

$$f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n+1)!}=1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}+\cdots+\frac{(-1)^n x^{2n}}{(2n+1)!}+\cdots$$ for all real numbers x.

(a) Find $f'(0)$ and $f''(0)$. Determine whether f has a local maximum, a local minimum, or neither at $x = 0$. Give a reason for your answer.

(b) Show that $1-\frac{1}{3!}$ approximates $f(1)$ with error less than $\frac{1}{100}$.

(c) Show that $y=f(x)$ is a solution to the differential equation $xy'+y=\cos x$.

29. (2003B BC6) The function f has a Taylor series about $x = 2$ that converges to $f(x)$ for all x in the interval of convergence. The nth derivative of f at $x = 2$ is given by $f^{(n)}(2)=\frac{(n+1)!}{3^n}$ for $n \ge 1$, and $f(2) = 1$.

(a) Write the first four terms and the general term of the Taylor series for f about $x = 2$.

(b) Find the radius of convergence for the Taylor series for f about $x = 2$. Show the work that leads to your answer.

(c) Let g be a function satisfying $g(2) = 3$ and $g'(x)=f(x)$ for all x. Write the first four terms and the general tem of the Taylor series for g about $x = 2$.

(d) Does the Taylor series for g as defined in part (c) converge at $x = -2$? Give a reason for your answer.

AP Calculus BC
Chapter 9 AP Exam Problems

30. (2004 BC6) Let *f* be the function given by $f(x) = \sin\left(5x + \frac{\pi}{4}\right)$, and let $P(x)$ be the third-degree Taylor polynomial for *f* about $x = 0$.

(a) Find $P(x)$.
(b) Find the coefficient of x^{22} in the Taylor series for *f* about $x = 0$.
(c) Use the Lagrange error bound to show that $\left|f\left(\frac{1}{10}\right) - P\left(\frac{1}{10}\right)\right| < \frac{1}{100}$.
(d) Let *G* be the function given by $G(x) = \int_0^x f(t)\,dt$. Write the third-degree Taylor polynomial for *G* about $x = 0$.

31. (2004B BC2) Let *f* be a function having derivatives of all orders for all real numbers. The third-degree Taylor polynomial for *f* about $x = 2$ is given by $T(x) = 7 - 9(x-2)^2 - 3(x-2)^3$.

(a) Find $f(2)$ and $f''(2)$.
(b) Is there enough information given to determine whether *f* has a critical point at $x = 2$? If not, explain why not. If so, determine whether $f(2)$ is a relative maximum, a relative minimum, or neither, and justify your answer.
(c) Use $T(x)$ to find an approximation for $f(0)$. Is there enough information given to determine whether *f* has a critical point at $x = 0$? If not, explain why not. If so, determine whether $f(0)$ is a relative maximum, a relative minimum, or neither, and justify your answer.
(d) The fourth derivative of *f* satisfies the inequality $\left|f^{(4)}(x)\right| \le 6$ for all *x* in the closed interval [0, 2]. Use the Lagrange error bound on the approximation to $f(0)$ found in part (c) to explain why $f(0)$ is negative.

32. (2005 BC6) Let *f* by a function with derivatives of all orders and for which $f(2) = 7$. When *n* is odd, the *n*th derivative of *f* at $x = 2$ is 0. When *n* is even and $n \ge 2$, the *n*th derivative of *f* at $x = 2$ is given by $f^{(n)}(2) = \frac{(n-1)!}{3^n}$.

(a) Write the sixth-degree Taylor polynomial for *f* about $x = 2$.
(b) In the Taylor series for *f* about $x = 2$, what is the coefficient of $(x-2)^{2n}$ for $n \ge 1$?
(c) Find the interval of convergence of the Taylor series for *f* about $x = 2$. Show the work that leads to your answer.

AP Calculus BC
Chapter 9 AP Exam Problems

33. (2005B BC3) The Taylor series about $x = 0$ for a certain function f converges to $f(x)$ for all x in the interval of convergence. The nth derivative of f at $x = 0$ is given by $f^{(n)}(0)=\frac{(-1)^{n+1}(n+1)!}{5^n(n-1)^2}$ for $n\geq 2$. The graph of f has a horizontal tangent line at $x = 0$, and $f(0) = 6$.

(a) Determine whether f has a relative maximum, a relative minimum, or neither at $x = 0$. Justify your answer.
(b) Write the third-degree Taylor polynomial for f about $x = 0$.
(c) Find the radius of convergence of the Taylor series for f about $x = 0$. Show the work that leads to your answer.

34. (2006 BC6) The function f is defined by the power series:
$f(x)=-\frac{x}{2}+\frac{2x^2}{3}-\frac{3x^3}{4}+\cdots+\frac{(-1)^n nx^n}{n+1}+\cdots$ for all real numbers x for which the series converges. The function g is defined by the power series:
$g(x)=1-\frac{x}{2!}+\frac{x^2}{4!}-\frac{x^3}{6!}+\cdots+\frac{(-1)^n x^n}{(2n)!}+\cdots$ for all real numbers x for which the series converges.

(a) Find the interval of convergence of the power series for f. Justify your answer.
(b) The graph of $y=f(x)-g(x)$ passes through the point $(0,-1)$. Find $y'(0)$ and $y''(0)$. Determine whether y has a relative minimum, a relative maximum, or neither at $x = 0$. Give a reason for your answer.

35. (2006B BC6) The function f is defined by $f(x)=\frac{1}{1+x^3}$. The Maclaurin series for f is given by $1-x^3+x^6-x^9+\cdots+(-1)^n x^{3n}+\cdots$, which converges to $f(x)$ for $-1<x<1$.

(a) Find the first three nonzero terms and the general term for the Maclaurin series for $f'(x)$.
(b) Use your results fron part (a) to find the sum of the infinite series $-\frac{3}{2^2}+\frac{6}{2^5}-\frac{9}{2^8}+\cdots+(-1)^n\frac{3n}{2^{3n-1}}+\cdots$.
(c) Find the first four nonzero terms and the general term for the Maclaurin series representing $\int_0^{1/2} f(t)dt$.
(d) Use the first three nonzero terms of the infinite series found in part (c) to approximate $\int_0^{1/2} f(t)dt$. What are the properties of the terms of the series representing $\int_0^{1/2} f(t)dt$ that guarantee that this approximation is within $\frac{1}{10,000}$ of the exact value of the integral?

36. (2007 BC6) Let *f* be the function given by $f(x)=e^{-x^2}$.

(a) Write the first four nonzero terms and the general term of the Taylor series for f about $x = 0$.

(b) Use your answer to part (b) to find $\lim_{x\to 0}\frac{1-x^2-f(x)}{x^4}$.

(c) Write the first four nonzero terms of the Taylor series for $\int_0^x e^{-t^2}dt$ about $x = 0$. Use the first two terms of your answer to estimate $\int_0^{1/2} e^{-t^2}dt$.

(d) Explain why the estimate found in part (c) differs from the actual value of $\int_0^{1/2} e^{-t^2}dt$ by less than $\frac{1}{200}$.

37. (2007B BC6) Let *f* be the function given by $f(x)=6e^{-x/3}$ for all *x*.

(a) Find the first four nonzero terms and the general term for the Taylor series for *f* about $x = 0$.

(b) Let g be the function given by $g(x)=\int_0^x f(t)dt$. Find the first four nonzero terms and the general term for the Taylor series for g about $x = 0$.

(c) The function *h* satisfies $h(x)=kf'(ax)$ for all *x*, where *a* and *k* are constants. The Taylor series for *h* about $x = 0$ is given by $h(x)=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^n}{n!}+\cdots$. Find the values of *a* and *k*.

Answer Key

1. A	1993	BC	#31	46%
2. C	1985	BC	#14	82%
3. A	1988	BC	#44	35%
4. A	1993	BC	#16	57%
5. B	1998	BC	#18	35%
6. A	1998	BC	#22	68%
7. D	1998	BC	#76	60%
8. A	1985	BC	#10	49%
9. E	1985	BC	#42	64%
10. B	1988	BC	#13	77%
11. A	1993	BC	#43	26%
12. E	1998	BC	#14	68%
13. D	1998	BC	#27	35%
14. A	1998	BC	#89	56%
15. D	1985	BC	#31	53%
16. C	1988	BC	#38	52%
17. C	1993	BC	#27	49%
18. B	1998	BC	#84	40%