More Series

Anton 11.10

Important Maclaurin Series: memorize

Interval of conv.?

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \qquad (-\infty, \infty)$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots \qquad (-\infty, \infty)$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots \qquad (-\infty, \infty)$$

→

Important Maclaurin Series: memorize

Interval of conv.?

$$\frac{1}{1-x}=\sum_{n=0}^{\infty} x^n = 1+x+x^2+x^3+\cdots \qquad (-1,1)$$

$$\frac{1}{1+x}=\sum_{n=0}^{\infty} (-1)^n x^n = 1-x+x^2-x^3+\cdots \qquad (-1,1)$$

$$\ln(1+x)=\sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^n}{n} = x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots \qquad (-1,1]$$

→

Important Maclaurin Series: memorize

Interval of conv.?

$$\tan^{-1} x=\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} = x-\frac{x^3}{3}+\frac{x^5}{5}-\cdots \qquad [-1,1]$$

The following you do NOT need to memorize (prepare to have your mind blown.)

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Find the series for: e^{ix}

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \cdots$$

$$e^{ix} = 1 + (ix) + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \frac{(ix)^6}{6!} + \frac{(ix)^7}{7!} + \cdots$$

$$= 1 + ix - \frac{x^2}{2!} - \frac{ix^3}{3!} + \frac{x^4}{4!} + \frac{ix^5}{5!} - \frac{x^6}{6!} - \frac{ix^7}{7!} + \frac{x^8}{8!} + \cdots$$

$$= \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} \cdots\right) + i\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right)$$

$$e^{ix} = \cos x + i \sin x$$

→

Euler's Formula, or Euler's Identity

$$e^{ix} = \cos x + i \sin x$$

Using the special case when $x = \pi$

$$e^{i\pi} = -1$$

$$\boxed{e^{i\pi} + 1 = 0}$$

→

AP Packet #21 (1995 BC4)

a) $f(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \cdots$

$P_2 = 3 + -2(x-1) + (x-1)^2$

$f(.7) \approx 3 + -2(-.3) + (-.3)^2 = 3.69$

b) $P_3 = 3 + -2(x-1) + (x-1)^2 + \frac{4}{3!}(x-1)^3$

$f(1.2) \approx 3 - 2(.2) + (.2)^2 + \frac{2}{3}(.2)^3 = 2.645$

c) $f' = -2 + 2(x-1) + \frac{4 \cdot 3}{3!}(x-1)^2 \Big|_{x=1.2} = -2 + 2(.2) + 2(.2)^2$

$= -1.52$

→

AP Packet #32 (2005 BC6)

a) $P_6 = f(2) + \frac{f''(2)}{2!}(x-2)^2 + \frac{f^{(4)}(2)}{4!}(x-2)^4 + \frac{f^{(6)}(2)}{6!}(x-2)^6$

$= 7 + \frac{\frac{1!}{3^2}(x-2)^2}{2!} + \frac{\frac{3!}{3^4}(x-2)^4}{4!} + \frac{\frac{5!}{3^6}(x-2)^6}{6!} + \cdots \frac{\frac{(2n-1)!}{3^{2n}}(x-2)^{2n}}{(2n)!}$

b) - - - - - - - - - - - - - - → $= \boxed{\frac{1}{3^{2n} \cdot 2n}}(x-2)^{2n}$

c) $\lim_{n\to\infty} \left| \frac{(x-2)^{2(n+1)}}{3^{2(n+1)} \cdot 2(n+1)} \cdot \frac{3^{2n} \cdot 2n}{(x-2)^{2n}} \right| = \left| \frac{(x-2)^2}{3^2} \right| < 1 \Rightarrow -1 < \frac{(x-2)^2}{9} < 1$

$-9 < (x-2)^2 < 9$

$-3 < x-2 < 3$

$-1 < x < 5$

$x = -1: \sum \frac{(-3)^{2n}}{3^{2n} \cdot 2n} \to \text{DIV}$

$x = 5: \sum \frac{(3)^{2n}}{3^{2n} \cdot 2n} \to \text{DIV}$

$\boxed{(-1, 5)}$

→

Homework:

Chapter 9 AP Packet
#9 – 17 odd, 22, 29