

Alternating Series

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Tests for Convergence/Divergence (so far):

1. Geometric Series Test
2. P-series Test (included harmonic series)
3. Divergence Test
4. Integral Test
5. Ratio Test
6. Root Test
7. Limit Comparison Test
8. Comparison Test



Informal Principle #1

Constant terms in the denominator can be ignored without affecting the convergence or divergence of a series.

$$\sum_{k=1}^{\infty} \frac{1}{2^k + 1}$$

BEHAVES LIKE GEOMETRIC

$$w/ r = 1/2$$

$$\Rightarrow \sum_{k=1}^{\infty} \frac{1}{2^{k+1}} \text{ CONV}$$

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k} - 2}$$

BEHAVES LIKE P-SERIES

$$p = 1/2$$

$$\Rightarrow \sum_{k=1}^{\infty} \frac{1}{\sqrt{k} - 2} \text{ DIV}$$

$$\sum_{k=1}^{\infty} \frac{1}{\left(k + \frac{1}{2}\right)^3}$$

BEHAVES LIKE P-SERIES, $p = 3$

$$\sum_{k=1}^{\infty} \frac{1}{(k + 1/2)^3} \text{ CONV}$$



Informal Principle #2

Highest powers of k matter the most in a polynomial. Ignoring the rest will not affect the convergence or divergence of the series.

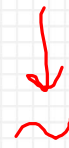
$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k^3 + 2k}}$$

BEHAVE LIKE P-SERIES, $p = 3/2$

$$\sum \frac{1}{\sqrt{k^3 + 2k}} \quad \underline{\text{CONV.}}$$

$$\sum_{k=1}^{\infty} \frac{6k^4 - 2k^3 + 1}{k^5 + k^2 - 2k}$$

"BEHAVES LIKE"



$$\sum_{k=1}^{\infty} \frac{6}{k}$$

BEHAVES LIKE A HARMONIC \Rightarrow DIVERGES.



Alternating Series

$$a_k > 0$$

In general, an alternating series has one of the following two forms:

$$\sum_{k=1}^{\infty} (-1)^{k+1} a_k = a_1 - a_2 + a_3 - a_4 + \dots$$

$$\sum_{k=1}^{\infty} (-1)^k a_k = -a_1 + a_2 - a_3 + a_4 - \dots$$

In both cases, assume a_k is positive.

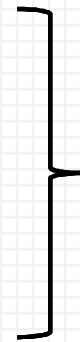


Alternating Series Test

An alternating series converges if the following are both true:

1. $a_1 > a_2 > a_3 > \dots$

2. $\lim_{k \rightarrow \infty} a_k = 0$



MAGNITUDE

The absolute value
of the terms
(disregard sign) are
decreasing to 0.



$$\text{Ex: } \sum_{k=1}^{\infty} \frac{(-1)^k}{k}$$

ALTERNATING
HARMONIC

$$= -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \dots$$

THE MAGNITUDE OF TERMS ARE
DECREASING TO 0 \Rightarrow CONVERGES.

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{k} \quad \underline{\underline{\text{CONV}}}$$

$$\text{Ex: } \sum_{k=1}^{\infty} \frac{(-1)^{k+1} (k+3)}{k(k+1)}$$

$$= \frac{4}{1(2)} - \frac{5}{2(3)} + \frac{6}{3(4)} - \frac{7}{4(5)} + \dots$$

THE MAG. OF TERMS ARE
DECREASING TO 0

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1} (k+3)}{k(k+1)} \quad \underline{\underline{\text{CONV}}}$$



Absolute Convergence:

An alternating series will converge *absolutely* if:

$$\sum_{k=1}^{\infty} |(-1)^k a_k| = a_1 + a_2 + a_3 + \dots \text{ converges.}$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{2^k} \rightarrow \text{CONV ABSOLUTELY}$$

$$\sum \left| \frac{(-1)^k}{2^k} \right| = \sum \frac{1}{2^k} \text{ GEOM w/ } r=1/2 \rightarrow \text{CONV}$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{k} \rightarrow \text{DOES NOT CONV ABSOLUTELY}$$

$$\sum \left| \frac{(-1)^k}{k} \right| = \sum \frac{1}{k} \rightarrow \text{DIVERGES.}$$



Theorem: If a series converges absolutely, then it converges (two for one.)

Conditional Convergence:

If an alternating series converges but the series of absolute values does not, then the original alternating series converges *conditionally*.

Ex: $\sum_{k=1}^{\infty} \frac{(-1)^k}{k}$ CMV CONDITIONALLY



Ratio Test for Absolute Convergence

Let $\sum u_k$ have nonzero terms.

Let $\rho = \lim_{k \rightarrow \infty} \left| \frac{u_{k+1}}{u_k} \right| = \lim_{k \rightarrow \infty} \left| u_{k+1} \cdot \frac{1}{u_k} \right|$

1. If $\rho < 1$ then the series converges *absolutely*.
2. If $\rho > 1$ then the series diverges.
3. If $\rho = 1$ then the test is inconclusive*.

*Further examination is required.



$$\text{Ex: } \sum_{k=1}^{\infty} (-1)^k \frac{2^k}{k!}$$

$$\rho = \lim_{k \rightarrow \infty} \left| \frac{u_{k+1}}{u_k} \right|$$

$$= \lim_{k \rightarrow \infty} \left| \frac{2^{k+1}}{(k+1)!} \cdot \frac{k!}{2^k} \right|$$

$$= 0 < 1$$

$$\therefore \sum (-1)^k \frac{2^k}{k!} \text{ CMV } \underline{\text{ABSOLUTELY}}$$

$$\left(\sum \frac{2^k}{k!} \right)$$



Tests for Convergence/Divergence (so far):

1. Geometric Series Test $\text{CMV } |r| < 1$
2. P-series Test (including harmonic) $\text{CMV } p > 1$
3. Divergence Test
4. Integral Test
5. Ratio Test for Absolute Convergence CMV ABSOLUTELY
6. Root Test (for Absolute Convergence) $p < 1$
- ~~7.~~ Limit Comparison Test
- ~~8.~~ Comparison Test
9. Alternating Series Test



Homework:

Anton 11.7 # 1 – 29 odd

