Agenda

- HW Review: AP FRQ Packet 2010 #1,2
- CW/HW: AP FRQ Packet 2010 #3,4
- Return Chapter 10 Test 2 ?
- Embark on a new journey…

Sequences

Anton 11.1

Objectives

- Given a sequence in **closed** form, write out the terms
- Given a sequence in **expanded** form, write in closed form
- Given a sequence, determine if it **converges/diverges**

A **sequence** $\{a_n\}$ is a listing of values of a_n as n goes from 1 to ∞.

$$\{a_n\} = a_1, a_2, a_3, \ldots$$

GENERAL TERM

Ex: $\{2^n\} = 2^1, 2^2, 2^3, \ldots$

ALTERNATING SEQUENCE.

Ex: $\left\{(-1)^n \frac{x^n}{n!}\right\} = -\frac{x}{1!}, \frac{x^2}{2!}, -\frac{x^3}{3!}, \ldots$

Express in bracket notation:

$n=1 \quad n=2 \quad n=3$

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots \qquad \left\{\frac{n}{n+1}\right\}$

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots \qquad \left\{\frac{1}{2^n}\right\}$

$1, -3, 5, -7, \ldots \qquad \left\{(-1)^{n+1}(2n-1)\right\}$

Limits of Sequences

The sequence $\{a_n\} \xrightarrow{\text{CONVERGES TO}} L$ if $\lim\limits_{n\to\infty} a_n = L$

The sequence $\{a_n\}$ diverges if $\lim\limits_{n\to\infty} a_n$ diverges.

Find the limit of the following sequences.

$\left\{\frac{1}{n}\right\} \to 0 \qquad \lim\limits_{n\to\infty} \frac{1}{n} = 0$

$\{n+1\} \to$ DIVERGES $\qquad \lim\limits_{n\to\infty} n+1 = \infty$

$\left\{(-1)^{n+1}\right\} = 1, -1, 1, -1, 1, \ldots \qquad \lim\limits_{n\to\infty} (-1)^{n+1} = \text{DNE}$

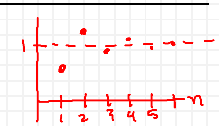
$\to$ DIVERGES

Find the limit of the following sequences.

$\left\{\frac{n}{n+1}\right\} \to 1 \qquad \lim\limits_{n\to\infty} \frac{n}{n+1} \overset{L}{\Rightarrow} \frac{1}{1}$

DISCRETE MATH

$\left\{1+\left(-\frac{1}{2}\right)^n\right\} \to 1 \qquad \lim\limits_{n\to\infty} 1+\left(-\frac{1}{2}\right)^n = 1$



$\left\{\frac{n}{2n+1}\right\} \to 1/2$

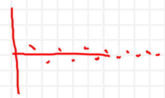
Find the limit of the following sequences.

$\left\{(-1)^{n+1}\dfrac{n}{2n+1}\right\} \to$ DIVERGES

POSITIVE TERMS CONV TO 1/2

NEG. TERMS CONV. TO −1/2

$\left\{(-1)^{n+1}\dfrac{1}{n}\right\} \to 0$



$\{8-2n\}$

Find the limit of the following sequences.

$\left\{\dfrac{n}{e^n}\right\} \to 0$

$\lim\limits_{n\to\infty} \dfrac{n}{e^n} \to 0$

$\boxed{\left\{\sqrt[n]{n}\right\} \to 1}$

$n^{1/n}$

∞^0

$\lim\limits_{n\to\infty} n^{1/n} \Rightarrow y = n^{1/n}$

$\ln y = \frac{1}{n}\ln n \quad (0\cdot\infty)$

$= \dfrac{\ln n}{n} \quad \left(\frac{\infty}{\infty}\right)$

$\to \dfrac{1/n}{1} \Rightarrow \dfrac{1}{n} \to 0$

$\ln y \to 0 \Rightarrow y = e^0 = \boxed{1}$

Some other examples:

$\dfrac{1}{2}, \dfrac{1}{3}, \dfrac{1}{2^2}, \dfrac{1}{3^2}, \dfrac{1}{2^3}, \dfrac{1}{3^3}, \ldots$

$1, \dfrac{1}{2}, 1, \dfrac{1}{3}, 1, \dfrac{1}{4}, \ldots$

Thrm: A sequence converges $\Leftrightarrow$ even numbered terms converge to L and the odd numbered terms converge to L.

Recursive Sequences – an example

$\{a_n\} \Rightarrow a_{n+1} = \dfrac{1}{2}\left(a_n + \dfrac{3}{a_n}\right) \qquad a_1 = 1$

List out the first few terms:

Do you think the sequence converges?

Recursive Sequences – an example

$$\{a_n\} \Rightarrow a_{n+1} = \frac{1}{2}\left(a_n + \frac{3}{a_n}\right) \qquad a_1 = 1$$

This sequence will converge if: $\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} a_n = L$

Closure

Q: Given a sequence, how do you determine whether it converges or diverges?

A: Take a limit!

Q: What's the difference between a **sequence** and a **series**?

A: Stay tuned…

Homework:

FRQ Packet 2010 #3,4
Anton 11.1 #1 – 31 odd