AP Calculus AB
(6.1) Area: Worksheet A

Name Key

You must show all work similar to class examples to receive credit. Please put ALL work on a separate sheet of paper. DO NOT squeeze your work on this side of the sheet NOR the back of this sheet – you will not receive any credit!

Calculate the area of the region bounded by the following:

1. $y = 4 - \frac{1}{3}x^2$ $y = 0$, $x = 0$, and $x = 3$
2. $y = 4x - x^2$, $y = 0$, $x = 1$, and $x = 3$
3. $y = x^2 - 2x - 3$, $y = 0$, $x = 0$, and $x = 2$
4. $y = \frac{1}{2}(x^2 - 10)$, $y = 0$, $x = -2$, and $x = 3$
5. $y = x^3$, $y = 0$, $x = -1$, and $x = 2$
6. $y = \sqrt[3]{x}$, $y = 0$, $x = -1$, and $x = 8$
7. $y = \sqrt{x - 4}$, $y = 0$, $x = 8$
8. $y = x^2 - 4x + 3$, and $x - y - 1 = 0$
9. $y = x^2$ and $y = x + 2$
10. $y = 2\sqrt{x}$, $y = 2x - 4$, and $x = 0$
11. $y = x^2 - 4x$, and $y = -x^2$
12. $y = x^2 - 2$ and $y = 2x^2 + x - 4$

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13. $x = 6y - y^2$ and $x = 0$
14. $x = -y^2 + y + 2$ and $x = 0$
15. $x = 4 - y^2$ and $x + y - 2 = 0$
16. $x = y^2 - 3y$ and $x - y + 3 = 0$
17. $y^2 - 2x = 0$ and $y^2 + 4x - 12 = 0$
18. $x = y^4$ and $x = 2 - y^4$

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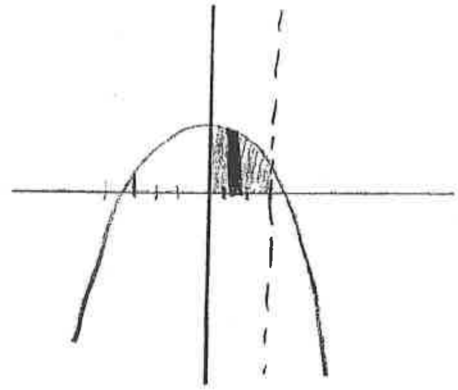
Worksheet Area of Region / Between Curves

1) $y = 4 - \frac{1}{3}x^2 \quad y=0 \quad x=0 \quad x=3$

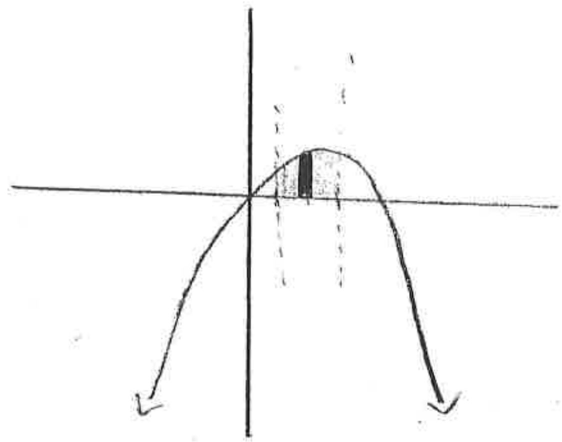


$$\int_0^3 \left(4-\frac{1}{3}x^2\right)dx$$

$$4x - \frac{1}{9}x^3 \Big]_0^3$$

$$\left[4(3)-\frac{1}{9}(3)^3\right] - [0-0] = 12-3 = \boxed{9}$$

2) $y = 4x - x^2 \quad y=0 \quad x=1 \quad x=3$

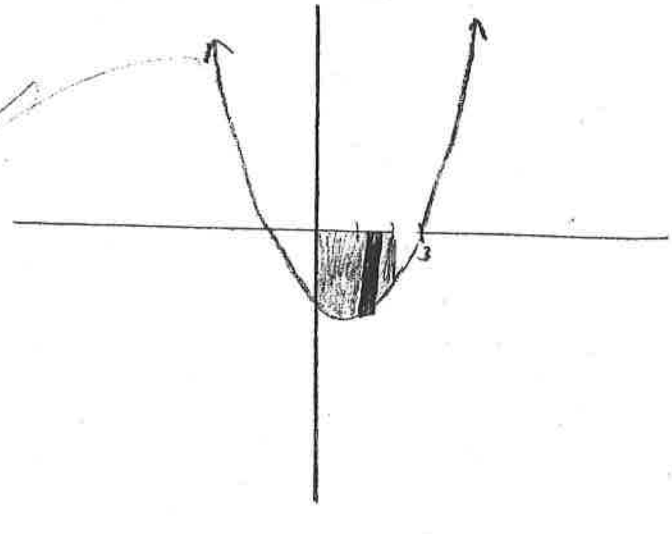


$$\int_1^3 (4x-x^2)\,dx$$

$$2x^2 - \frac{1}{3}x^3 \Big]_1^3$$

$$(18-9)-(2-1/3) = \frac{22}{3} \text{ or } 7.3\overline{3}$$

3) $y = x^2 - 2x - 3,\ y=0,\ x=0,\ x=2$



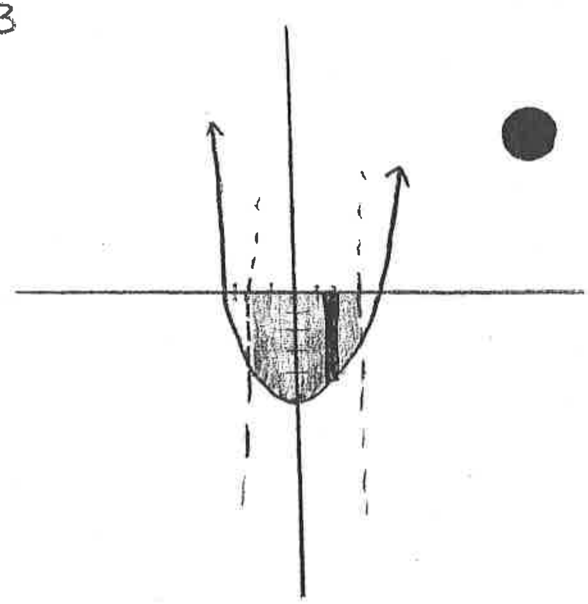
$$-\int_0^2 (x^2-2x-3)\,dx$$

$$-\left[\frac{x^3}{3} - x^2 - 3x\right]_0^2$$

$$-\left[\frac{8}{3}-4-6\right] - \cancel{[0]}$$

$$-[-7.33] = \boxed{\frac{22}{3}}$$

$y = \frac{1}{2}(x^2 - 10) \quad y = 0 \quad x = -2 \quad x = 3$



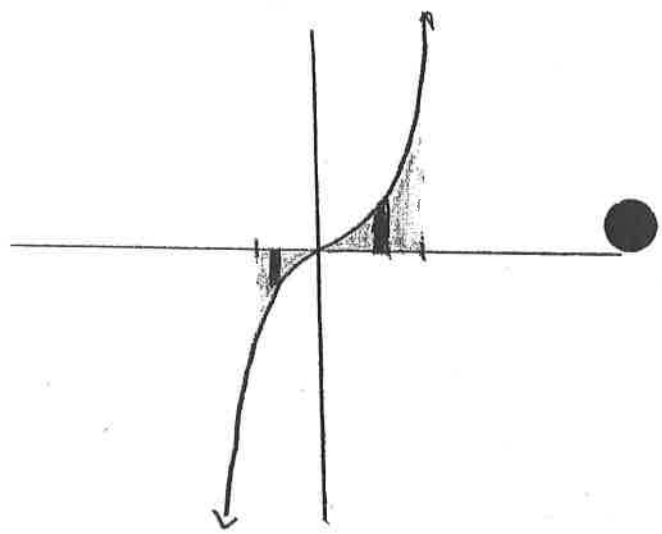
$- \int_{-2}^{3} \left(\frac{1}{2}x^2 - 5\right) dx$

$- \left[\frac{x^3}{6} - 5x\right]_{-2}^{3}$

$- \left[\frac{27}{6} - 15\right] - \left[-\frac{8}{6} + 10\right] = +19.167$

$\frac{115}{6}$

5) $y = x^3 \quad y = 0 \quad x = -1 \quad x = 2$



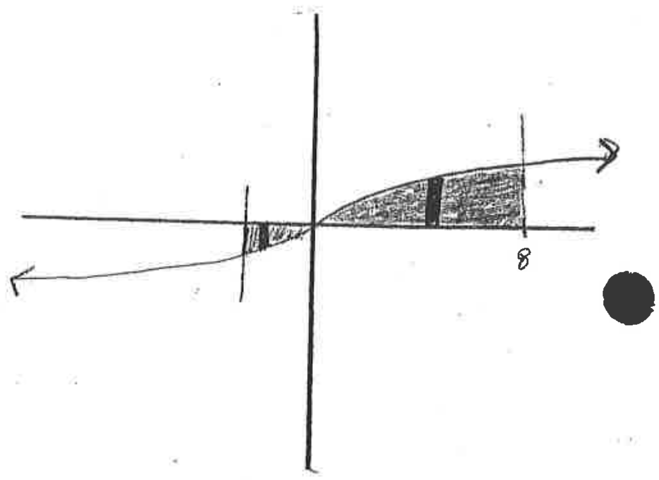
$- \int_{-1}^{0} x^3 dx + \int_{0}^{2} x^3 dx$

$- \left.\frac{x^4}{4}\right]_{-1}^{0} + \left.\frac{x^4}{4}\right]_{0}^{2}$

$- \left[0 - \frac{1}{4}\right] + \frac{16}{4}$

$\frac{1}{4} + 4 = \boxed{4.25}$

6) $y = \sqrt[3]{x} \quad y = 0 \quad x = -1 \quad x = 8$



$- \int_{-1}^{0} x^{1/3} dx + \int_{0}^{8} x^{1/3} dx$

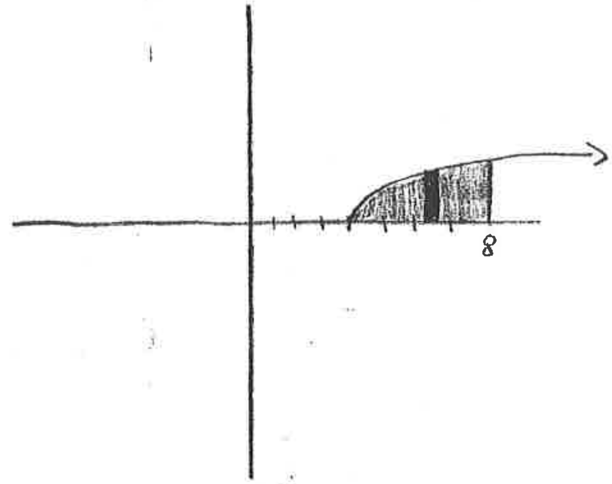
$\left[\frac{3x^{4/3}}{4}\right]_{-1}^{0} + \left[\frac{3}{4}x^{4/3}\right]_{0}^{8}$

$+ \left[0 + \frac{3}{4}\right] + \left[12 - 0\right] = \boxed{12.75}$ $12\frac{3}{4}$

$\frac{51}{4}$

7

$y = \sqrt{x-4} \quad y = 0 \quad x = 8$



$$\int_4^8 \sqrt{x-4}\, dx$$

$u = x - 4$

$du = dx$

$$\int u^{1/2}\, du$$

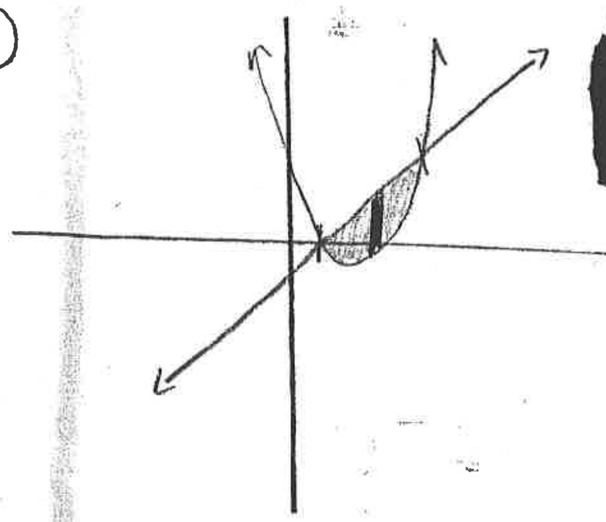
$$\frac{2}{3}u^{3/2} = \frac{2}{3}(x-4)^{3/2}\Big]_4^8$$

$$\frac{2}{3}\left[4^{3/2} - 0\right] = \frac{2}{3}\underset{64}{\sqrt{4^3}} = \frac{2}{3}(8) = \boxed{\frac{16}{3}}$$

8

$y = x^2 - 4x + 3 \qquad x - y - 1 = 0$

$y = x - 1$



Intersection Pts:

$x^2 - 4x + 3 = x - 1$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 1 \quad x = 4$

$$\int_1^4 \left[\overset{U}{(x-1)} - \overset{L}{(x^2 - 4x + 3)}\right] dx$$

$$\int_1^4 (x - 1 - x^2 + 4x - 3)\, dx$$

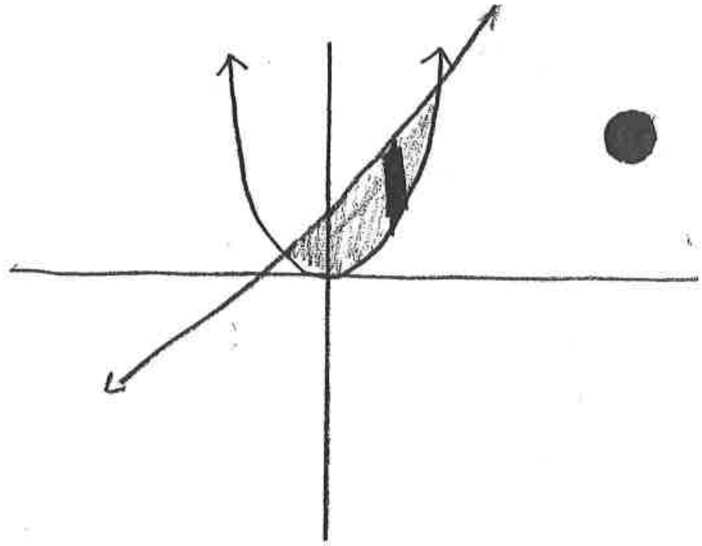
$$\int_1^4 -x^2 + 5x - 4 = 0$$

$$-\frac{x^3}{3} + \frac{5x^2}{2} - 4x\Big]_1^4$$

$$\left[-\frac{64}{3} + \frac{5(16)}{2} - 16\right] - \left[-\frac{1}{3} + \frac{5}{2} - 4\right]$$

$$\boxed{= 4.5}$$

$y = x^2 \qquad y = x + 2$



$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$

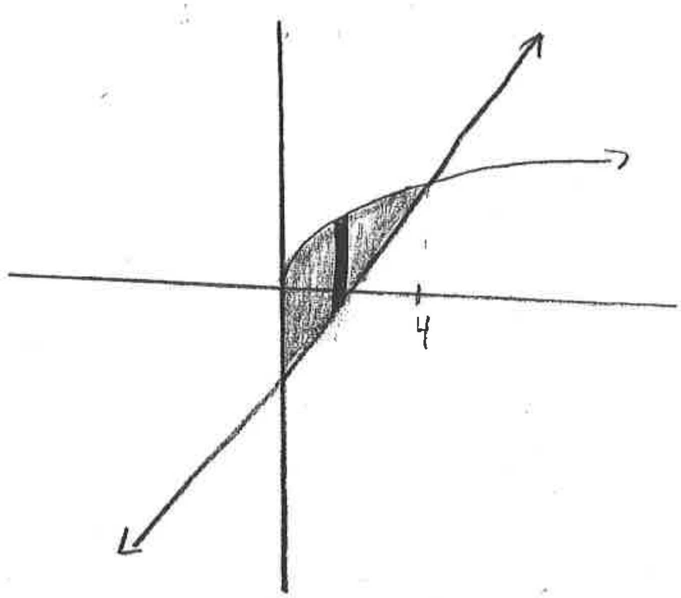
$\int_{-1}^{2} \left[(x+2) - (x^2)\right] dx$

$\int_{-1}^{2} (-x^2 + x + 2)\, dx$

$\left.\frac{-x^3}{3} + \frac{x^2}{2} + 2x\right]_{-1}^{2} =$

$\left[\frac{-8}{3} + 2 + 4\right] - \left[\frac{1}{3} + \frac{1}{2} - 2\right] = \boxed{4.5}$

$y = 2\sqrt{x} \qquad y = 2x - 4 \qquad x = 0$



$(2\sqrt{x})^2 = (2x-4)^2$

$4x = 4x^2 - 16x + 16$

$4x^2 - 20x + 16 = 0$

$4(x^2 - 5x + 4) = 0$

$4(x-4)(x-1) = 0$

$x = 4,\ x = 1\ \times$

$\sqrt{x} = x - 2$

$x = x^2 - 4x + 4$

$0 = x^2 - 5x + 4$

$0 = (x-4)(x-1)$

$x = 4$

$x = 1$

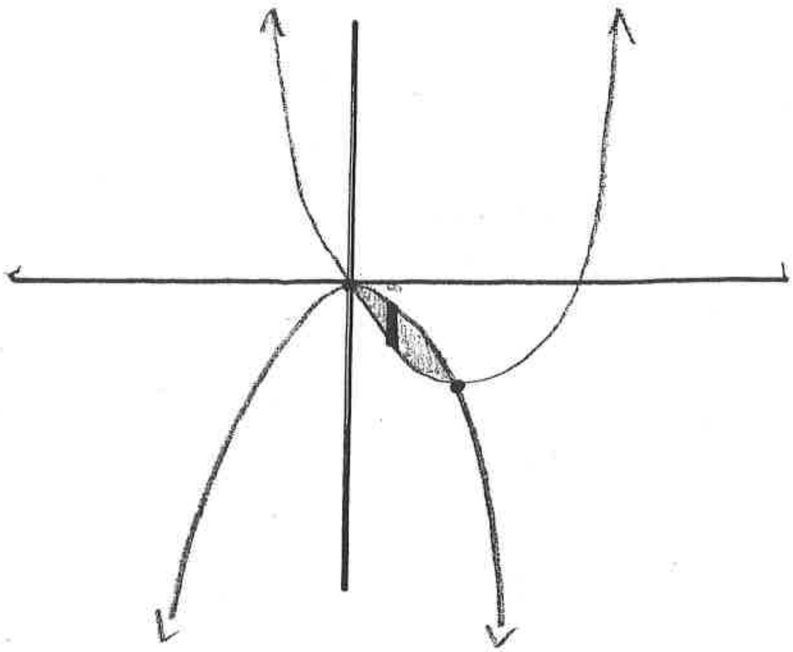
$\int_0^4 (2\sqrt{x}) - (2x - 4)\, dx$

$\int_0^4 (2x^{1/2} - 2x + 4)\, dx$

$\left.\frac{4}{3}x^{3/2} - x^2 + 4x\right]_0^4 = \left[\frac{32}{3} - 16 + 16\right] - [0] = \boxed{\frac{32}{3}}$

ea Worksheet 1 #11-18

) $y = x^2 - 4x$ ⇝ Lower
$y = -x^2$ ⇝ Upper



$x^2 - 4x = -x^2$

$2x^2 - 4x = 0$

$2x(x-2) = 0$

$x = 0$

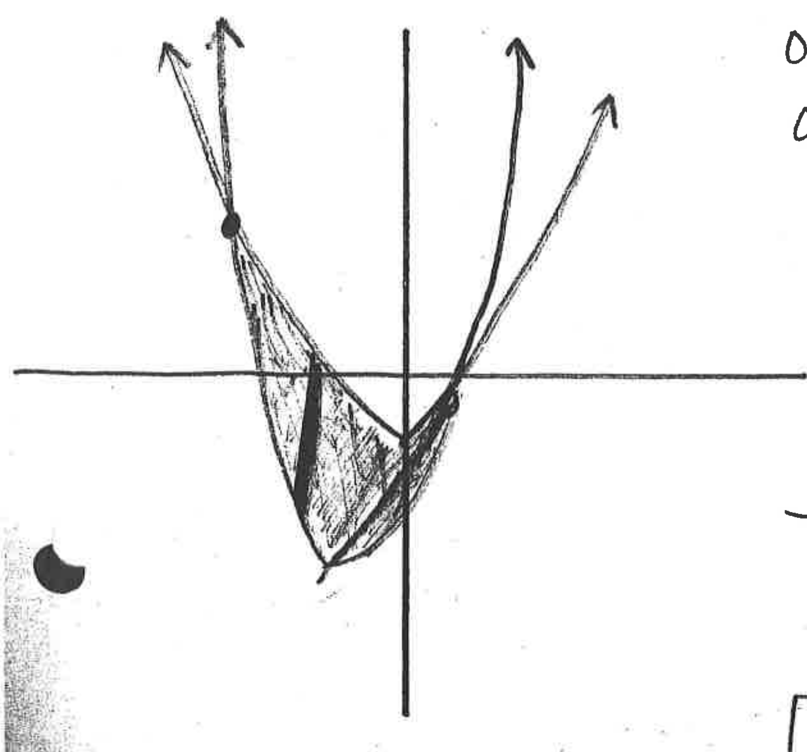
$x = 2$

$$\int_0^2 (-x^2 - x^2 + 4x)\,dx$$

$$\int_0^2 (-2x^2 + 4x)\,dx$$

$$\left.-\frac{2x^3}{3} + 2x^2\right]_0^2 = \boxed{\frac{8}{3} \approx 2.\overline{6}}$$

12) $y = x^2 - 2$ $y = 2x^2 + x - 4$



$x^2 - 2 = 2x^2 + x - 4$

$0 = x^2 + x - 2$

$0 = (x+2)(x-1)$

$x = -2$ $x = 1$

$$\int_{-2}^{1} \left[(x^2 - 2) - (2x^2 + x - 4)\right] dx$$

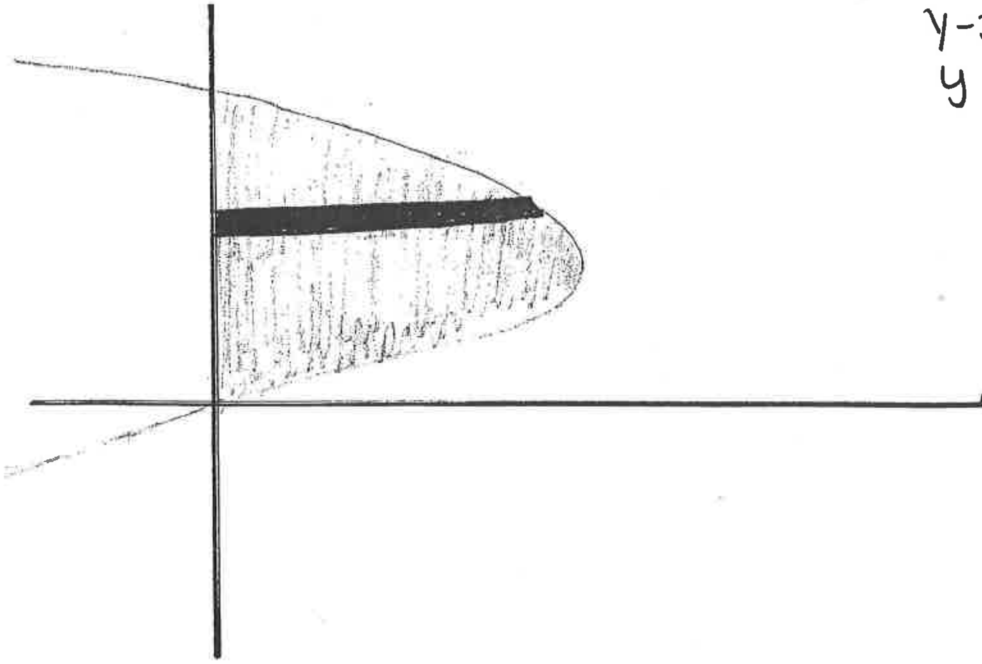
$$\int_{-2}^{1} (-x^2 - x + 2)\,dx$$

$$\left[\frac{-x^3}{3} - \frac{x^2}{2} + 2x\right]_{-2}^{1}$$

$\boxed{A = 4.5}$

$x = 6y - y^2$ y-limits $[0,6]$
$x = 0$

$y^2 - 6y = -x$
$y^2 - 6y + 9 = 9 - x$
$(y-3)^2 = 9 - x$
$y - 3 = \pm\sqrt{9-x}$
$y = 3 \pm \sqrt{9-x}$

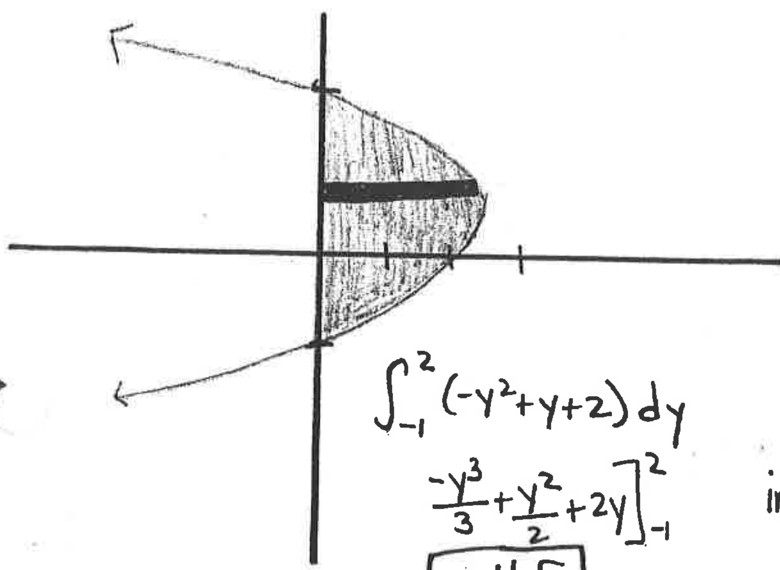


$$\int_0^6 (6y - y^2)\,dy$$

$$\left.\frac{6y^2}{2} - \frac{y^3}{3}\right]_0^6 = \left[3y^2 - \frac{y^3}{3}\right]_0^6 = \boxed{36}$$

14) $x = -y^2 + y + 2$ $x = 0$

$x = -y^2 + y + 2$
$y^2 - y = 2 - x$
$y^2 - y + \frac{1}{4} = -x + 2 + \frac{1}{4}$
$(y - 1/2)^2 = \frac{9}{4} - x$
$y - \frac{1}{2} = \pm\sqrt{9/4 - x}$
$y = \frac{1}{2} \pm \sqrt{9/4 - x}$



$$\int_{-1}^{2} (-y^2 + y + 2)\,dy$$

$$\left.\frac{-y^3}{3} + \frac{y^2}{2} + 2y\right]_{-1}^{2}$$

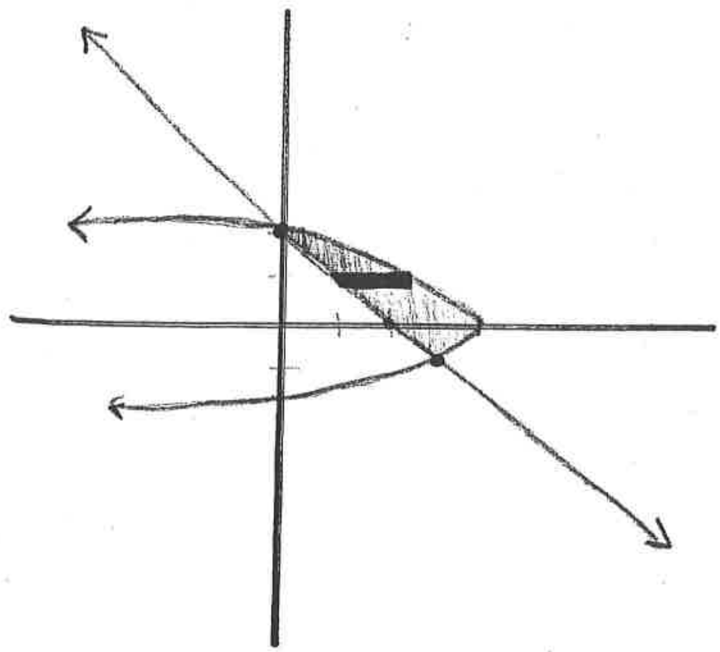
$\boxed{= 4.5}$

intercepts: $-y^2 + y + 2 = 0$ $\boxed{y = 2, -1}$
$y^2 - y - 2 = 0$
$(y-2)(y+1) = 0$

$4-y^2 \to y=\pm\sqrt{4-x}$

$x+y-2=0 \to y=-x+2$

$x=2-y$



Intersection pts:

$4-y^2=2-y$

$y^2-y-2=0$

$(y-2)(y+1)=0$

$y=2,-1$

$$A=\int_{-1}^{2}\left[(4-y^2)-(2-y)\right]dy$$

$$A=\int_{-1}^{2}(-y^2+y+2)\,dy$$

$$\left.\frac{-y^3}{3}+\frac{y^2}{2}+2y\right]_{-1}^{2}=\boxed{4.5}$$

16) $x=y^2-3y \qquad x-y+3=0$

$y=\pm\sqrt{x+9/4}+3/2 \qquad y=x+3$

Integrate:

$x=y^2-3y \qquad x=y-3$

Intersection pts:

$y^2-3y=y-3$

$y^2-4y+3=0$

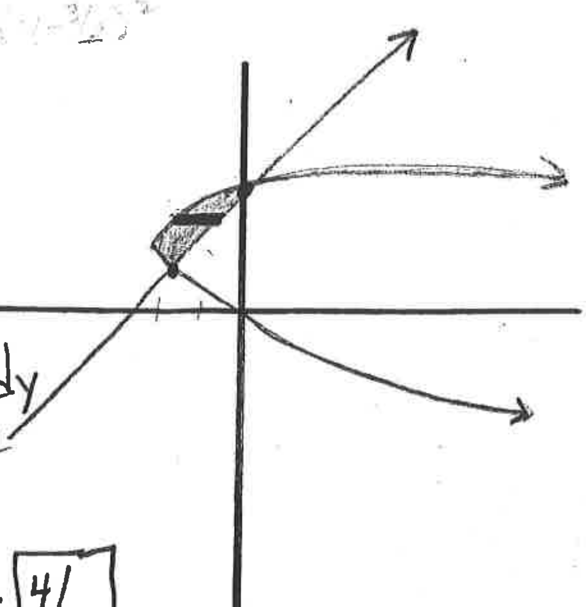
$(y-3)(y-1)=0$

$y=3$ and $y=1$

$$\int_1^3\left[(y-3)-(y^2-3y)\right]dy$$

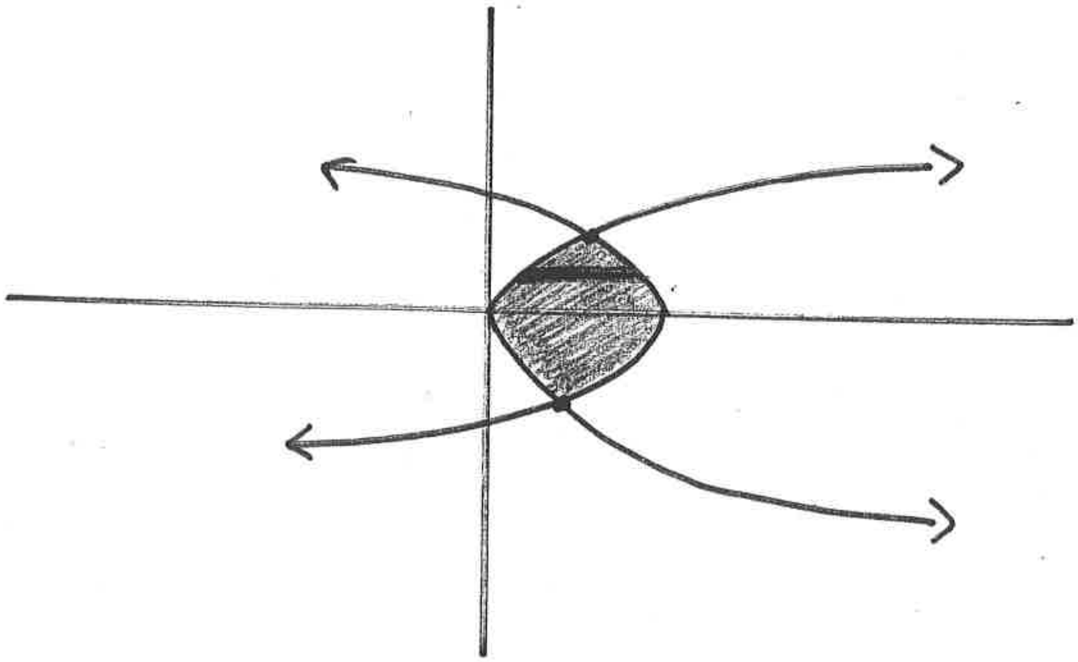
$$\int_1^3(-y^2+4y-3)\,dy$$

$$\left[-\frac{y^3}{3}+2y^2-3y\right]_1^3=\boxed{4/3}$$



$y^2 - 2X = 0 \longrightarrow y = \pm\sqrt{2x} \longrightarrow X = \frac{1}{2}y^2$

$y^2 + 4x - 12 = 0 \longrightarrow y = \pm\sqrt{12-4x} \longrightarrow x = -\frac{1}{4}y^2 + 3$



$2X = -4X + 12$

$6X - 12 = 0$

$6(x-2) = 0$

$X = 2 \quad (2,2)$

$\quad\quad\quad (2,-2)$

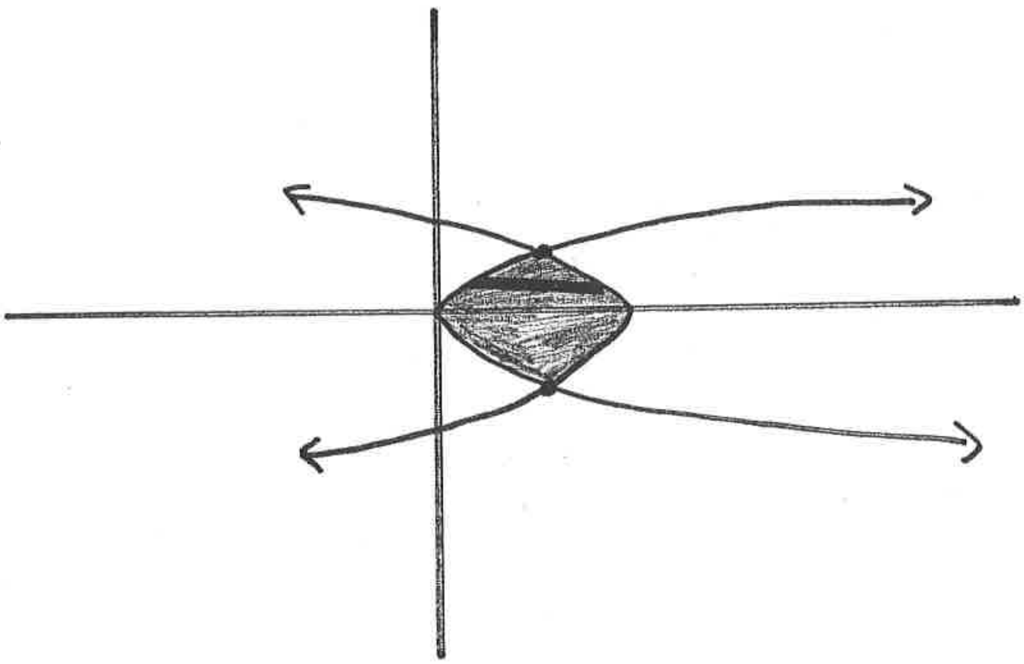
$$\int_{-2}^{2} \left[\left(-\tfrac{1}{4}y^2 + 3\right) - \left(\tfrac{1}{2}y^2\right) \right] dy$$

$$\int_{-2}^{2} \left(-\frac{3}{4}y^2 + 3\right) dy$$

$$\left[-\frac{1}{4}y^3 + 3y \right]_{-2}^{2} = \boxed{8}$$

$x = y^4 \rightarrow y = \pm \sqrt[4]{x}$

$x = 2 - y^4 \rightarrow y = \pm \sqrt[4]{2-x}$



$y^4 = 2 - y^4$

$2y^4 = 2$

$y^4 = 1$

$y = \pm 1$

$$\int_{-1}^{1} \left[(2-y^4) - (y^4)\right] dy$$

$$\int_{-1}^{1} (2 - 2y^4)\, dy$$

$$\left[2y - \frac{2y^5}{5}\right]_{-1}^{1} = \boxed{\frac{16}{5} \text{ or } 3.2}$$