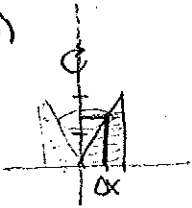


Shell Method p460 1-21 odd

#1) 

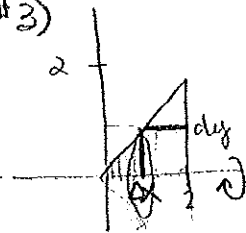
$$V = \int 2\pi(R)(H)(TH)$$

$$V = \int_0^1 2\pi(x)(2x) dx$$

$$= 4\pi \int_0^1 x^2 dx$$

$$= 4\pi \left[\frac{x^3}{3} \right]_0^1$$

$$= \boxed{\frac{4}{3}\pi}$$

#3) 

$$V = \int 2\pi(R)(H)(TH)$$

$$= \int_0^2 2\pi(y)(2-y) dy$$

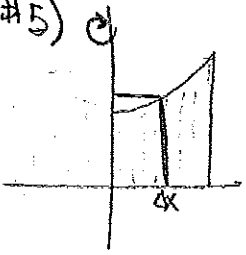
$$= 2\pi \int_0^2 (2y - y^2) dy$$

$$= 2\pi \left[y^2 - \frac{y^3}{3} \right]_0^2$$

$$= 2\pi \left[4 - \frac{8}{3} \right] - 0$$

$$= \boxed{\frac{8}{3}\pi}$$

$r = y$
 $h = 2 - y$

#5) 

$$V = \int 2\pi(R)(H) dx$$

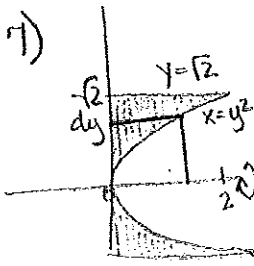
$$= 2\pi \int_0^2 (x) \left(1 + \frac{x^2}{4} \right) dx$$

$$= 2\pi \int_0^2 x + \frac{x^3}{4} dx$$

$$= 2\pi \left[\frac{x^2}{2} + \frac{x^4}{16} \right]_0^2$$

$$= 2\pi (2 + 1) - 0$$

$$= \boxed{6\pi}$$

#7) 

$$V = \int 2\pi(R)(H)(dy)$$

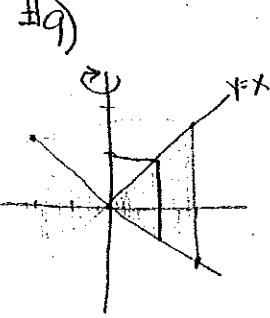
$$= \int_0^{\sqrt{2}} 2\pi(y)(y^2) dy$$

$$= 2\pi \int_0^{\sqrt{2}} y^3 dy$$

$$= 2\pi \left[\frac{y^4}{4} \right]_0^{\sqrt{2}}$$

$$= 2\pi \left[\frac{4}{4} \right]$$

$$= \boxed{2\pi}$$

#9) 

$$V = \int 2\pi(R)(H)(dx)$$

$$= \int_0^2 2\pi(x) \left(x + \frac{1}{2}x \right) dx$$

$$= 2\pi \int_0^2 x^2 + \frac{x^2}{2} dx$$

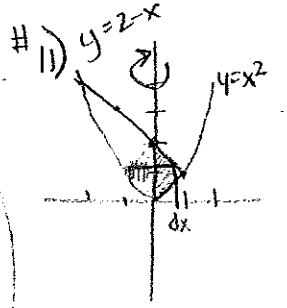
$$= 2\pi \left[\frac{x^3}{3} + \frac{x^3}{6} \right]_0^2$$

$$= 2\pi \left[\frac{8}{3} + \frac{8}{6} \right] - 0$$

$$= 2\pi \left[\frac{16}{6} + \frac{8}{6} = \frac{24}{6} = 4 \right]$$

$$= \boxed{8\pi}$$

OR
 $(x)(\frac{3}{2}x)$
 $\int \frac{3}{2}x^2$
 $2\pi \cdot \frac{x^3}{\frac{2}{3}} \Big|_0^2$
 $= 8\pi$

#11) 

$$V = \int 2\pi(R)(H)(TH)$$

$$= \int_0^1 2\pi(x)(2-x-x^2) dx$$

$$= 2\pi \int_0^1 (2x - x^2 - x^3) dx$$

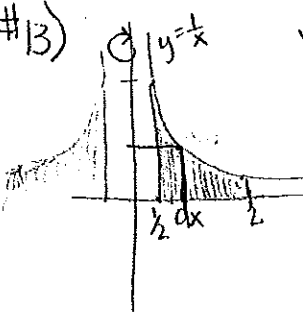
$$= 2\pi \left[x^2 - \frac{x^3}{3} - \frac{x^4}{4} \right]_0^1$$

$$= 2\pi \left[1 - \frac{1}{3} - \frac{1}{4} \right] - 0$$

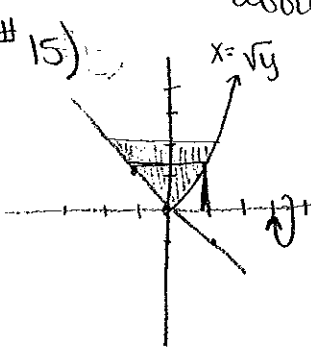
$$= 2\pi \left(\frac{5}{12} \right)$$

$$= \boxed{\frac{5}{6}\pi}$$

Top
Bottom

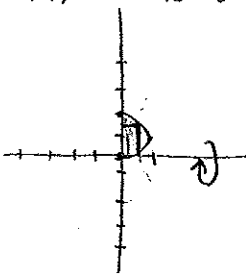
#13) 

$$\begin{aligned}
 V &= \int 2\pi (R)(H)(TH) \\
 &= 2\pi \int x \left(\frac{1}{x}\right) dx \\
 &= 2\pi \int_{\frac{1}{2}}^2 1 dx \\
 &= 2\pi \left[x\right]_{\frac{1}{2}}^2 \\
 &= 2\pi \left(2 - \frac{1}{2}\right) = \frac{3}{2} \\
 &= \boxed{3\pi}
 \end{aligned}$$

#15)  about x-axis

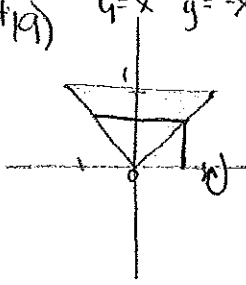
$$\begin{aligned}
 V &= \int 2\pi (R)(H)(TH) dy \\
 &= 2\pi \int_0^2 (y)(\sqrt{y} + y) dy \\
 &= 2\pi \int_0^2 y^{3/2} + y^2 dy \\
 &= 2\pi \left[\frac{2}{5} y^{5/2} + \frac{y^3}{3} \right]_0^2 \\
 &= 2\pi \left[\frac{2}{5} \cdot \frac{(\sqrt{2})^5}{2 \cdot \sqrt{2}} + \frac{8}{3} \right] - 0 \\
 &= 2\pi \left[\frac{8\sqrt{2}}{5} + \frac{8}{3} \right] \\
 &= 2\pi \cdot 8 \cdot \frac{1}{15} [3\sqrt{2} + 5] \\
 &= \boxed{\frac{16}{15}\pi [3\sqrt{2} + 5]}
 \end{aligned}$$

#17) $x = 2y - y^2$ $x = 0$



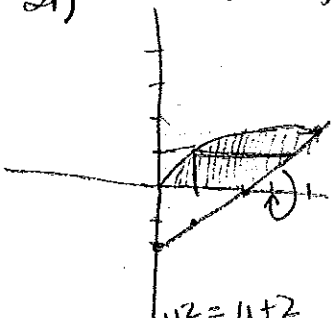
$$\begin{aligned}
 V &= \int 2\pi (R)(H)(TH) \\
 &= \int_0^2 2\pi (y)(2y - y^2) dy \\
 &= 2\pi \int_0^2 2y^2 - y^3 dy \\
 &= 2\pi \left[\frac{2}{3} y^3 - \frac{y^4}{4} \right]_0^2 \\
 &= 2\pi \left[\frac{16}{3} - 4 \right] \\
 &= 2\pi \left[\frac{4}{3} \right] \\
 &= \boxed{\frac{8}{3}\pi}
 \end{aligned}$$

#19) $y = x$ $y = -x$ $y = 1$



$$\begin{aligned}
 V &= \int 2\pi (R)(H)(TH) dy \\
 &= 2\pi \int_0^1 (y)(y + (-y)) dy \\
 &= 4\pi \int_0^1 y^2 dy \\
 &= 4\pi \left[\frac{y^3}{3} \right]_0^1 \\
 &= 4\pi \left[\frac{1}{3} - 0 \right] \\
 &= \boxed{\frac{4}{3}\pi}
 \end{aligned}$$

#21) $y = \sqrt{x}$, $y = 0$, $y = \frac{x-2}{x}$



$$\begin{aligned}
 V &= \int 2\pi (R)(H)(TH) dy \\
 &= \int_0^2 (y)(y+2-y^2) dy \\
 &= 2\pi \int_0^2 y^2 + 2y - y^3 dy \\
 &= 2\pi \left[\frac{y^3}{3} + y^2 - \frac{y^4}{4} \right]_0^2 \\
 &= 2\pi \left(\frac{8}{3} + 4 - 4 \right) \\
 &= \boxed{\frac{16}{3}\pi}
 \end{aligned}$$

$y^2 = y + 2$
 $y^2 - y - 2 = 0$
 $(y-2)(y+1) = 0$
 $y = 2, -1$