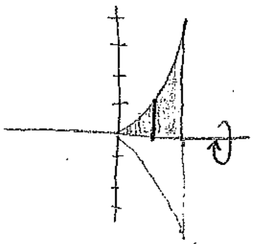
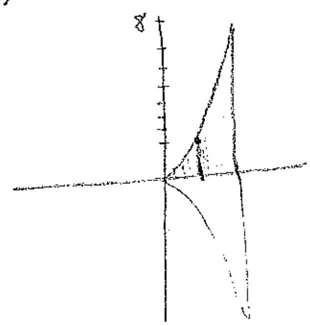
p453 9 - 12; , 25 - 30

9) $y = x^2$, $y = 0$, $x = 2$ (x-axis)



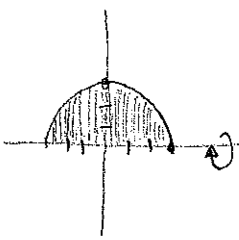
$$V = \int_0^2 \pi (x^2)^2 \, dx \quad (\pi R^2)$$
$$= \pi \int_0^2 x^4 \, dx$$
$$= \pi \left[\frac{x^5}{5}\right]_0^2$$
$$= \boxed{\frac{32}{5}\pi}$$

10) $y = x^3$, $y = 0$, $x = 2$ (x-axis)



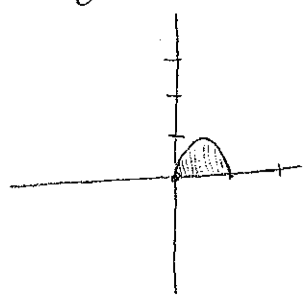
$$V = \int_0^2 \pi (x^3)^2 \, dx$$
$$= \pi \int_0^2 x^6 \, dx$$
$$= \pi \left[\frac{x^7}{7}\right]_0^2$$
$$= \boxed{\frac{128}{5}\pi}$$

11) $y = \sqrt{9 - x^2}$, $y = 0$ (x-axis)



$$V = \int_0^3 \pi \left[(9 - x^2)^{1/2}\right]^2 dx$$
$$= \pi \int_0^3 9 - x^2 \, dx$$
$$= \pi \left[9x - \frac{x^3}{3}\right]_0^3$$
$$= \pi (27 - 9) - 0$$
$$= \boxed{18\pi} \times 2 = \boxed{36\pi}$$

12) $y = x - x^2$, $y = 0$ about (x-axis)

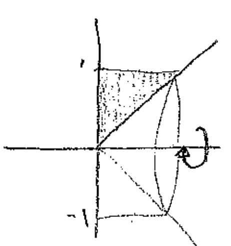


$x - x^2 = 0$
$x(1 - x) = 0$
$x = 0, 1$

$$V = \int_0^1 \pi (x - x^2)^2 \, dx$$
$$= \pi \int_0^1 x^2 - 2x^3 + x^4 \, dx$$
$$= \pi \left[\frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5}\right]_0^1$$
$$= \pi \left[\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right] - 0$$
$$= \boxed{\frac{\pi}{30}}$$

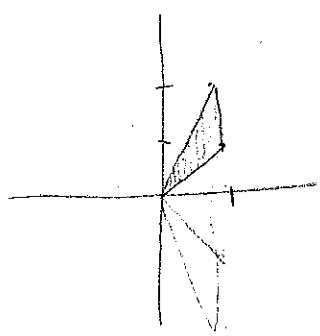
(X- AXIS)

#25) $y = x$; $y = 1$, $x = 0$ (x-axis)



$$V = \int_0^1 \pi \left((1)^2 - (x)^2\right) dx \quad (R(x)^2 - r(x)^2)\,dx$$
$$= \pi \int_0^1 1 - x^2 \, dx$$
$$= \pi \left[x - \frac{x^3}{3}\right]_0^1$$
$$= \pi \left(1 - \frac{1}{3}\right) - 0$$
$$= \boxed{\frac{2}{3}\pi}$$

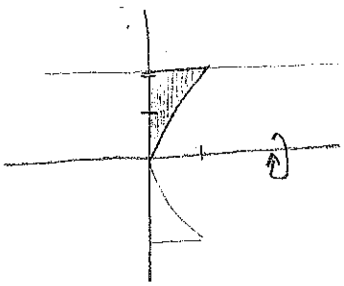
#26) $y = 2x$, $y = x$, $x = 1$ about x-axis



$2x = x$
$x = 0$

$$\pi \left(f(x)^2 - g(x)^2\right) dx$$
$$V = \int_0^1 \pi \left[(2x)^2 - (x)^2\right] dx$$
$$= \pi \int 4x^2 - x^2 \, dx$$
$$= \pi \int_0^1 3x^2 \, dx$$
$$= \pi \left[x^3\right]_0^1$$
$$= \boxed{\pi}$$

#27) $y = 2\sqrt{x}$, $y = 2$, $x = 0$ (x-axis)



$$V = \pi \int_0^1 [(2)^2 - (2\sqrt{x})^2] dx$$

$$= \pi \int_0^1 4 - 4x \, dx$$

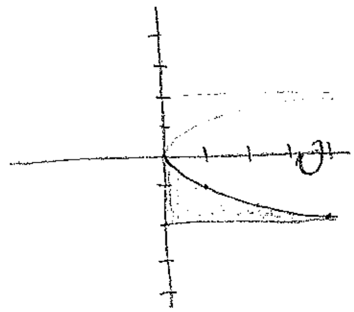
$$= \pi [4x - 2x^2]_0^1$$

$$= \pi [4 - 2] - 0$$

$$= \boxed{2\pi}$$

#28) $y = -\sqrt{x}$, $y = -2$, $x = 0$

$(R(x)^2 - r(x)^2) dx$



$$V = \int_0^4 \pi [(-2)^2 - (-\sqrt{x})^2] dx$$

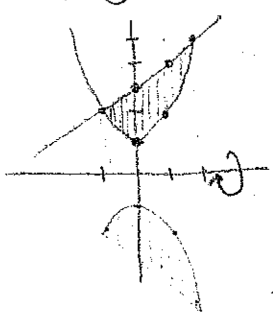
$$= \pi \int_0^4 4 - x \, dx$$

$$= \pi \left[4x - \frac{x^2}{2}\right]_0^4$$

$$= \pi (16 - 8) - 0$$

$$= \boxed{8\pi}$$

#29) $y = x^2 + 1$ $y = x + 3$ (x-axis)



$R(x)^2 - r(x)^2$

$$V = \int_{-1}^{2} \pi [(x+3)^2 - (x^2+1)^2] dx$$

$$= \pi \int_{-1}^{2} (x^2 + 6x + 9) - (x^4 + 2x^2 + 1)^2 dx$$

$$= \pi \int_{-1}^{2} x^2 + 6x + 9 - x^4 - 2x^2 - 1 \, dx$$

$$= \pi \int_{-1}^{2} -x^4 - x^2 + 6x + 8 \, dx$$

$$= \pi \left[-\frac{x^5}{5} - \frac{x^3}{3} + 3x^2 + 8x\right]_{-1}^{2}$$

$x^2 + 1 = x + 3$

$x^2 - x - 2 = 0$

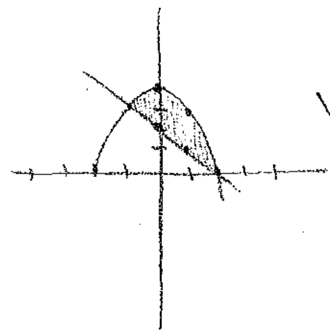
$(x - 2)(x + 1) = 0$

$$= \pi \left[-\frac{32}{5} - \frac{8}{3} + 12 + 16\right] - \left[\frac{1}{5} + \frac{1}{3} + 3 - 8\right]$$

$$= \pi \left[\frac{284}{15}\right] + \left[+\frac{67}{15}\right]$$

$$= \frac{351}{15}\pi = \boxed{\frac{117}{5}\pi}$$

#30) $y = 4 - x^2$, $y = 2 - x$ (x-axis)



$$V = \int_{-1}^{2} \pi (4 - x^2)^2 - (2 - x)^2 dx$$

$$V = \pi \int_{-1}^{2} (16 - 8x^2 + x^4) - (4 - 4x + x^2) dx$$

$16 - 8x^2 + x^4 - 4 + 4x - x^2$

$$= \pi \int_{-1}^{2} x^4 - 9x^2 + 4x + 12 \, dx$$

$$= \pi \left[\frac{x^5}{5} - 3x^3 + 2x^2 + 12x\right]_{-1}^{2}$$

$$= \pi \left(\frac{32}{5} - 24 + 8 + 24\right) - \left(-\frac{1}{5} + 3 + 2 - 12\right)$$

$$= \pi \left(\frac{72}{5}\right) + \left(+\frac{36}{5}\right)$$

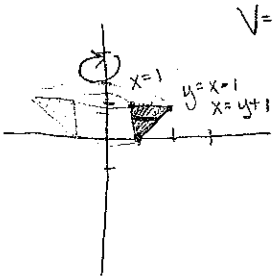
$$= \boxed{\frac{108}{5}\pi}$$

p453 35 - 40 (Y → AXIS)

35) Δ w/ vertices (1,0), (2,1), (1,1)

$R(y)^2 - r(y)^2\, dy$

$V = \int_0^1 \pi\, (y+1)^2 - (1)^2\, dy$



$= \pi \int_0^1 (y^2 + 2y + 1 - 1\, dy$

$= \pi \int_0^1 y^2 + 2y\, dy$

$= \pi \left[\frac{y^3}{3} + y^2 \right]_0^1$

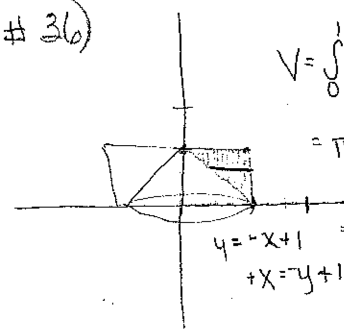
$= \pi \left(\frac{1}{3} + 1 \right) - 0$

$= \boxed{\frac{4}{3}\pi}$

\# 36) Δ (0,1), (1,0)(1,1) about y-axis

$R(x)^2 - r(x)^2\, dx$

$V = \int_0^1 \pi \left[(1)^2 - (1-y)^2 \right] dy$



$= \pi \int_0^1 1 - (1 - 2y + y^2)\, dy$

$\quad -1 + 2y - y^2$

$y = -x + 1$

$+x = -y + 1$

$= \pi \int_0^1 2y - y^2\, dy$

$= \pi \left[y^2 - \frac{y^3}{3} \right]_0^1$

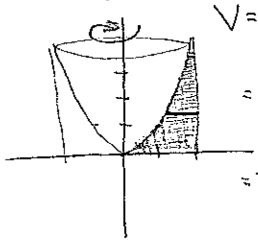
$= \pi \left(1 - \frac{1}{3} \right) - 0$

$= \boxed{\frac{2}{3}\pi}$

\#37) $y = x^2$, x-axis, $x=2$

$\pm\sqrt{y} = x$

$V = \int_0^4 \pi \left[(2)^2 - (\sqrt{y})^2\, dy \right.$



$= \pi \int_0^4 4 - y\, dy$

$= \pi \left[4y - \frac{y^2}{2} \right]_0^4$

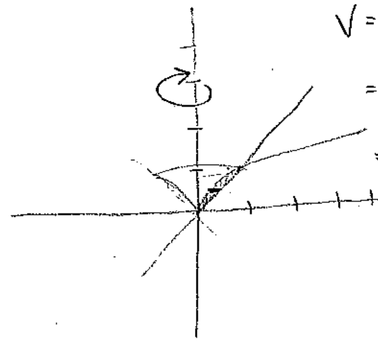
$= \pi \left[16 - 8 \right]$

$= \boxed{8\pi}$

\#38) $y = \sqrt{x}$, below $y = x$ line

$R(y)^2 - r(y)^2\, dy$

$V = \int_0^1 \pi\, (y)^2 - (y^2)^2\, dy$



$= \pi \int_0^1 y^2 - y^4\, dy$

$= \pi \left(\frac{y^3}{3} - \frac{y^5}{5} \right) \Big|_0^1$

$= \pi \left(\frac{1}{3} - \frac{1}{5} \right) - 0$

$\frac{5}{15} - \frac{3}{15}$

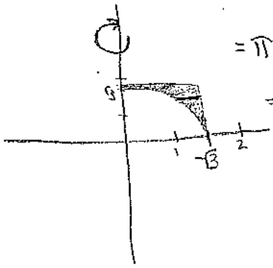
$= \boxed{\frac{2}{15}\pi}$

\#39) $x^2 + y^2 = 3$ $\quad x = \sqrt{3}$

$x = \sqrt{3 - y^2}$ $\quad y = \sqrt{3}$

$Ry^2 - ry^2\, dy$

$V = \int_0^{\sqrt{3}} \pi\, (\sqrt{3})^2 - (\sqrt{3 - y^2})^2\, dy$



$= \pi \int_0^{\sqrt{3}} 3 - (3 - y^2)$

$= \pi \int_0^{\sqrt{3}} y^2\, dy$

$= \pi \left[\frac{y^3}{3} \right]_0^{\sqrt{3}}$

$= \pi \frac{3\sqrt{3}}{3}$

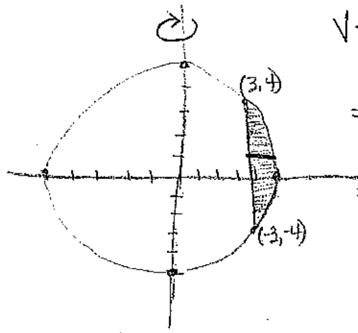
$= \sqrt{3}\pi$

\# 40) $x = 4$, $x^2 + y^2 = 25$

$x = \pm\sqrt{25 - y^2}$

$Ry^2 - ry^2\, dy$

$V = \int_{-3}^{3} \pi \left[(25 - y^2)^{1/2} \right]^2 - (4)^2\, dy$



$= \pi \int_{-3}^{3} (25 - y^2) - 16\, dy$

$= \pi \int_{-3}^{3} 9 - y^2\, dy$

$= \pi \left[9y - \frac{y^3}{3} \right]_{-3}^{3}$

$= \pi\, (27 - 9) - (-27 + 9)$

$= \pi \left[18 + +18 \right]$

$= \boxed{36\pi}$

$x^2 + y^2 = 25$

$(4)^2 + y^2 = 25$

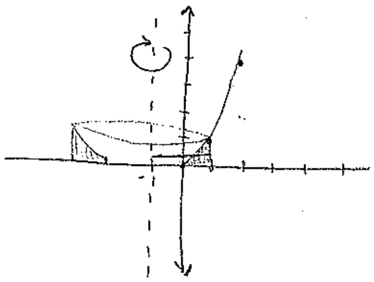
$16 + y^2 = 25$

$y^2 = 9$

$y = \pm 3$

p454 Washers

#41) $y = x^2$, $x = \pm\sqrt{y}$; below x-axis, right by $x = 1$, about line $x = -1$



$$R_y^2 - r_y^2 \; dy$$

$$V = \int_0^1 \pi\left[(1-(-1))^2 - (\sqrt{y} - (-1))^2\right] dy$$

($1+1 = 2^2$, $\sqrt{y}+1$)

$$= \pi \int_0^1 4 - (y + 2\sqrt{y} + 1)\, dy$$

($-y - 2\sqrt{y} - 1$)

$$= \pi \int_0^1 3 - y - 2y^{1/2}\, dy$$

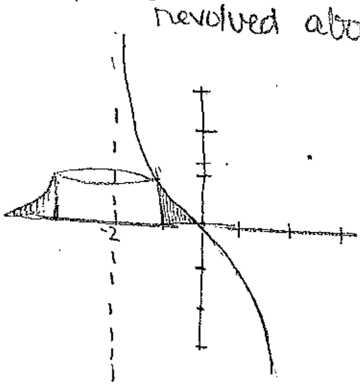
$$= \pi\left[3y - \frac{y^2}{2} - \frac{4}{3}y^{3/2}\right]_0^1$$

$$= \pi\left[3 - \frac{1}{2} - \frac{4}{3}\right] - 0$$

$$= \boxed{\frac{7}{6}\pi}$$

$\sqrt[3]{-y} = x^3$

#42) $y = -x^3$, x-axis, $x = -1$
revolved about $x = -2$



$$R_y^2 - r_y^2 \; dy$$

$$V = \int_0^1 \pi\left((-y)^{1/3} - (-2)\right)^2 - (-1 + (+2))^2\, dy$$

$$= \pi \int_0^1 (-y^{1/3} + 2)^2 - (1)^2\, dy$$

$$= \pi \int_0^1 (y^{2/3} - 4y^{1/3} + 4) - 1$$

$$= \pi \int_0^1 y^{2/3} - 4y^{1/3} + 3\, dy$$

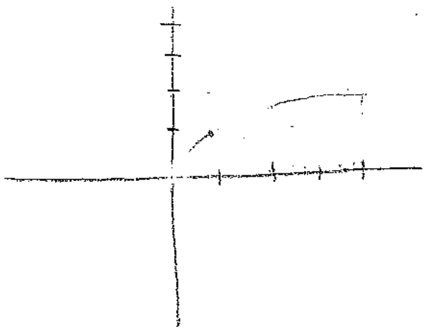
$$= \pi\left[\frac{3}{5}y^{5/3} - 3y^{4/3} + 3y\right]_0^1$$

$$= \pi\left(\frac{3}{5} - 3 + 3\right) - 0$$

$$= \boxed{\frac{3}{5}\pi}$$

p 454   47-48-49

#47) $y=\sqrt{x}$, $y=2$ and $x=0$



a) revolve about x-axis

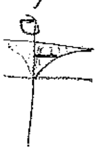
$$V=\int_0^4 \pi\left((2)^2-(\sqrt{x})^2\right)dx \quad \pi\, R(x)^2 - r(x)^2\, dx$$

$$=\pi\int_0^4 4-x\,dx$$

$$=\pi\left[4x-\frac{x^2}{2}\right]_0^4 = \pi\left(16-\frac{16}{2}\right)$$

$$=\boxed{8\pi}$$

b) revolve about y-axis



$$V=\int_0^2 \pi\left[y^2\right]^2 dy \quad Ry^2 \quad dy$$

$$=\pi\int_0^2 y^4\,dy$$

$$=\pi\left[\frac{y^5}{5}\right]_0^2$$

$$=\boxed{\frac{32}{5}\pi}$$

c) revolve about $y=2$

$$V=\int_0^4 \pi\,(2-\sqrt{x})^2\,dx \quad R(x)^2$$

$$=\pi\int_0^4 4-4\sqrt{x}+x\,dx \quad (4x^{1/2})$$

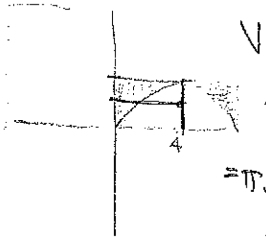
$$=\pi\left[4x-\frac{8}{3}x^{3/2}+\frac{x^2}{2}\right]_0^4$$

$$=\pi\left(16-\frac{8}{3}\cdot\frac{8}{1}+16\right)-0$$

$$\pi\quad 16-\frac{64}{3}+16$$

$$=\boxed{\frac{32}{3}\pi}$$

d) revolve about $x=4$



$$V=\int_0^2 \pi\,(4)^2-(4-y^2)^2\,dy \quad Ry^2-ry^2\,dy$$

$$=\pi\int_0^2 16-(16-8y^2+y^4)\,dy$$

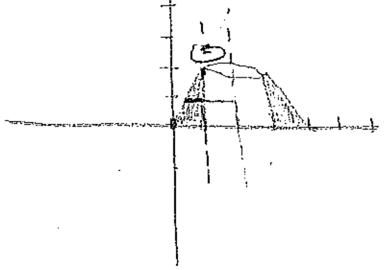
$$=\pi\int_0^2 +8y^2-y^4\,dy$$

$$=\pi\left[\frac{8}{3}y^3-\frac{y^5}{5}\right]_0^2$$

$$=\pi\left(\frac{64}{3}-\frac{32}{5}\right)-0$$

$$=\boxed{\frac{224}{15}\pi}$$

#48) $y=2x$, $y=0$ + $x=1$ ($x=\frac{y}{2}$)



a) revolve about $x=1$

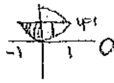
$$V=\int \pi\left(1-\frac{y}{2}\right)^2 dy \quad R(y)^2-ry^2\,dy$$

b) revolve about $x=2$

$$\int \pi\left[\left(2-\frac{y}{2}\right)^2-(2-1)^2\right]dy \quad R(y)^2 - Ry^2$$

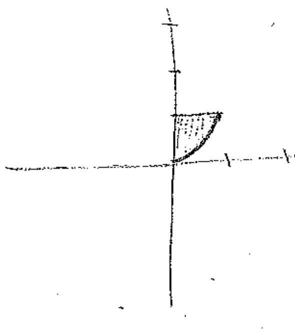
$$\pi\left[\left(2-\frac{y}{2}\right)^2-(1)^2\right]dy$$

#49) $y=x^2$ $y=1$



a) revolve $y=1$

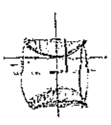
$$V=\int_{-1}^{1}\pi\,(1-x^2)^2\,dx$$



b) revolve $y=2$

$$V=\int_{-1}^{1}\pi\left[(2-x^2)^2-(2-1)^2\right]dx \quad R(x)^2-r(x)^2\,dx$$

$$\left[(2-x^2)^2-(1)^2\right]$$



c) revolve $y=-1$

$$V=\int_{-1}^{1}\pi\left[(1-(-1))^2-(x^2-(-1))^2\right)dx \quad R(x)^2-r(x)^2$$

$$(2)^2-(x^2+1)^2$$