Name: KEY

CALCULUS 2

WORKSHEET 5.3-2

Evaluate the following indefinite integrals by using substitution.

1. $\int \cos(2x-3)\,dx =$

$u = 2x-3 \quad du = 2\,dx \quad \frac{1}{2}du = dx$

$\int \cos u \cdot \frac{1}{2}\,du = \frac{1}{2}\int \cos u\,du = \frac{1}{2}\sin u + C$

$= \frac{1}{2}\sin(2x-3) + C$

2. $\int \sin 4x \cos 4x\,dx =$

$u = \sin 4x \quad du = 4\cos 4x\,dx \quad \frac{1}{4}du = \cos 4x\,dx$

$\int u \cdot \frac{1}{4}\,du = \frac{1}{4}\int u\,du = \frac{1}{4}\cdot\frac{1}{2}u^2 + C$

$= \frac{1}{8}\sin^2 4x + C$

3. $\int 2x^5 \sec^2(x^6+1)\,dx =$

$u = x^6+1 \quad du = 6x^5\,dx \quad \frac{1}{3}du = 2x^5\,dx$

$\int \sec^2 u \cdot \frac{1}{3}\,du = \frac{1}{3}\int \sec^2 u\,du = \frac{1}{3}\tan u + C$

$= \frac{1}{3}\tan(x^6+1) + C$

4. $\int \csc(3x+2)\cot(3x+2)\,dx =$

$u = 3x+2 \quad du = 3\,dx \quad \frac{1}{3}du = dx$

$\int \csc u \cot u \cdot \frac{1}{3}\,du = \frac{1}{3}\int \csc u \cot u\,du = -\frac{1}{3}\csc u + C$

$= -\frac{1}{3}\csc(3x+2) + C$

5. $\int \sec(3x^2)\tan(3x^2)x\,dx =$

$u = 3x^2 \quad du = 6x\,dx \quad \frac{1}{6}du = x\,dx$

$\int \sec u \tan u \cdot \frac{1}{6}\,du = \frac{1}{6}\int \sec u \tan u\,du = \frac{1}{6}\sec u + C$

$= \frac{1}{6}\sec(3x^2) + C$

6. $\int \frac{\cos x}{\sin^6 x}\,dx =$

$u = \sin x \quad du = \cos x\,dx$

$\int \frac{1}{u^6}\,du = -\frac{1}{5}u^{-5} + C = \frac{-1}{5u^5} + C$

$= -\frac{1}{5\sin^5 x} + C$

7. $\int \frac{\sin x}{\cos^4 x}\,dx =$

$u = \cos x \quad du = -\sin x\,dx \quad -du = \sin x\,dx$

$\int \frac{1}{u^4}\cdot -du = -\int \frac{1}{u^4}\,du = \frac{1}{3}u^{-3} + C$

$= \frac{1}{3\cos^3 x} + C$

8. $\int \frac{\cos(3x+1)}{\sin^2(3x+1)}\,dx =$

$u = \sin(3x+1) \quad du = 3\cos(3x+1)\,dx \quad \frac{1}{3}du = \cos(3x+1)\,dx$

$\int \frac{1}{u^2}\cdot\frac{1}{3}\,du = \frac{1}{3}\int \frac{1}{u^2}\,du = -\frac{1}{3}u^{-1} + C$

$= \frac{-1}{3\sin(3x+1)} + C$

9. $\int \frac{5\cos x}{(3-\sin x)^4}\,dx =$

$u = 3-\sin x \quad du = -\cos x\,dx \quad -5\,du = 5\cos x\,dx$

$\int \frac{1}{u^4}\cdot -5\,du = -5\int \frac{1}{u^4}\,du = -5\cdot -\frac{1}{3}u^{-3} + C$

$= \frac{5}{3(3-\sin x)^3} + C$

10. $\int (1+\sin 4x)^{1/2}\cos 4x\,dx =$

$u = 1+\sin 4x \quad du = 4\cos 4x\,dx \quad \frac{1}{4}du = \cos 4x\,dx$

$\int u^{\frac{1}{2}}\cdot\frac{1}{4}\,du = \frac{1}{4}\int u^{\frac{1}{2}}\,du = \frac{1}{4}\cdot\frac{2}{3}u^{\frac{3}{2}} + C$

$= \frac{1}{6}(1+\sin 4x)^{\frac{3}{2}} + C$

11. $\int \frac{\sec^2 3x}{\sqrt{\tan 3x}}\,dx =$

$u = \tan 3x \quad du = 3\sec^2 3x\,dx \quad \frac{1}{3}du = \sec^2 3x\,dx$

$\int \frac{1}{\sqrt{u}}\cdot\frac{1}{3}\,du = \frac{1}{3}\int \frac{1}{\sqrt{u}}\,du = \frac{1}{3}\cdot 2\,u^{\frac{1}{2}} + C$

$= \frac{2}{3}\sqrt{\tan 3x} + C$

12. $\int (1+\cos^3 x)\sin x\,dx =$

$u = \cos x \quad du = -\sin x\,dx \quad -du = \sin x\,dx$

$\int (1+u^3)\cdot -du = -\int (1+u^3)\,du = -u - \frac{1}{4}u^4 + C$

$-\cos x - \frac{1}{4}\cos^4 x + C$