CALCULUS 2

WORKSHEET 4.2-4.6-2

Name: KEY

Identify the intervals where f is increasing, decreasing, concave upward and concave downward. Then, find the coordinates of any local extreme points and points of inflection.

1. $f(x) = x^{\frac{6}{7}}$

$f'(x) = \frac{6}{7}x^{-\frac{1}{7}} = \frac{6}{7\sqrt[7]{x}}$

$x=0$

$f''(x) = -\frac{6}{49}x^{-\frac{8}{7}} = \frac{-6}{49\sqrt[7]{x^8}}$

$x=0$

		0	
f''	$-$		$-$
f'	$-$		$+$

(cusp) local min at $(0,0)$

incr: $(0,\infty)$

decr: $(-\infty,0)$

CD: $(-\infty,\infty)$

2. $f(x) = 3x^{\frac{1}{3}} - x$

$f'(x) = x^{-\frac{2}{3}} - 1 = \frac{1}{\sqrt[3]{x^2}} - 1$

$x=0$

$0 = \frac{1}{\sqrt[3]{x^2}} - 1$

$\sqrt[3]{x^2} = 1$

$x^2 = 1$

$x = \pm 1$

$f''(x) = -\frac{2}{3}x^{-\frac{5}{3}} = \frac{-2}{3\sqrt[3]{x^5}}$

$x=0$

		-1		0		1	
f''	$+$		$+$		$-$		$-$
f'	$-$		$+$		$+$		$-$

local min at $(-1,-2)$

PI at $(0,0)$

local max at $(1,2)$

incr: $(-1,1)$

decr: $(-\infty,-1)\cup(1,\infty)$

CU: $(-\infty,0)$

CD: $(0,\infty)$

3. $f(x) = x^{\frac{2}{3}}(x+5) = x^{\frac{5}{3}} + 5x^{\frac{2}{3}}$

$f'(x) = \frac{5}{3}x^{\frac{2}{3}} + \frac{10}{3}x^{-\frac{1}{3}}$

$0 = \frac{1}{3}x^{-\frac{1}{3}}(5x+10)$

$x=0,\ x=-2$

$f''(x) = \frac{10}{9}x^{-\frac{1}{3}} - \frac{10}{9}x^{-\frac{4}{3}}$

$0 = \frac{10}{9}x^{-\frac{4}{3}}(x-1)$

$x=0,\ x=1$

		-2		0		1	
f''	$-$		$-$		$-$		$+$
f'	$+$		$-$		$+$		$+$

local max at $(-2, 4.76)$

(cusp) local min at $(0,0)$

PI at $(1,6)$

incr: $(-\infty,-2)\cup(0,\infty)$

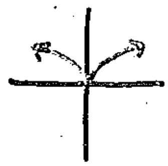
decr: $(-2,0)$

CU: $(1,\infty)$

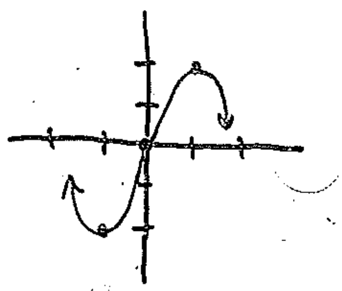
CD: $(-\infty,1)$

4.2-4.6-2

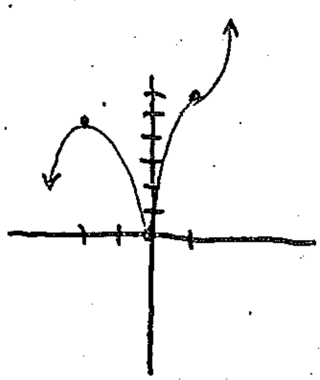
min: $(0,0)$
incr: $(0,\infty)$
decr: $(-\infty,0)$
CD: $(-\infty,\infty)$



min: $(-1,-2)$
max: $(1,2)$
incr: $(-1,1)$
decr: $(-\infty,-1),(1,\infty)$
PI: $(0,0)$
CU: $(-\infty,0)$
CD: $(0,\infty)$



max: $(-2,4.76)$
min: $(0,0)$
incr: $(-\infty,-2),(0,\infty)$
decr: $(-2,0)$
PI: $(1,6)$
CU: $(1,\infty)$
CD: $(-\infty,1)$



Name: KEY

CALCULUS 1

WORKSHEET 4.2-4.6-1

Identify the intervals where f is increasing, decreasing, concave upward and concave downward. Then, find the coordinates of any local extreme points and points of inflection.

1. $f(x) = x^3 - 3x + 2$

$f'(x) = 3x^2 - 3$ $\quad$ $f''(x) = 6x$

$x = \pm 1$ $\quad$ $x = 0$

f'' − − + +

f' + − − +

−1 0 1

local max: $(-1, 4)$

PI: $(0, 2)$

local min: $(1, 0)$

incr: $(-\infty, -1) \cup (1, \infty)$

decr: $(-1, 1)$

CU: $(0, \infty)$

CD: $(-\infty, 0)$

2. $f(x) = (x-5)^3 + 2$

$f'(x) = 3(x-5)^2$ $\quad$ $f''(x) = 6(x-5)$

$x = 5$ $\quad$ $x = 5$

f'' − +

f' + +

5

PI: $(5, 2)$

incr: $(-\infty, \infty)$

CU: $(5, \infty)$

CD: $(-\infty, 5)$

3. $f(x) = x^3 - \frac{3}{2}x^2 - 6x + 2$

$f'(x) = 3x^2 - 3x - 6$ $\quad$ $f''(x) = 6x - 3$

$0 = 3(x^2 - x - 2)$ $\quad$ $x = \frac{1}{2}$

$0 = 3(x-2)(x+1)$

$x = 2,\ x = -1$

f'' − − + +

f' + − − +

−1 $\frac{1}{2}$ 2

local max: $(-1, 5.5)$

PI: $(\frac{1}{2}, -1.25)$

local min: $(2, -8)$

incr: $(-\infty, -1) \cup (2, \infty)$

decr: $(-1, 2)$

CU: $(\frac{1}{2}, \infty)$

CD: $(-\infty, \frac{1}{2})$

4. $f(x) = 3x^4 - 4x^3$

$f'(x) = 12x^3 - 12x^2$ $\quad$ $f''(x) = 36x^2 - 24x$

$0 = 12x^2(x-1)$ $\quad$ $0 = 12x(3x-2)$

$x = 0,\ x = 1$ $\quad$ $x = 0,\ x = \frac{2}{3}$

f'' + − + +

f' − − − +

0 $\frac{2}{3}$ 1

PI: $(0, 0)$

PI: $(\frac{2}{3}, -.59)$

local min: $(1, -1)$

incr: $(1, \infty)$

decr: $(-\infty, 1)$

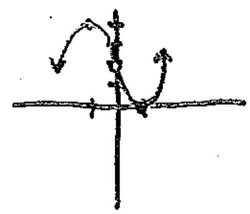
CU: $(-\infty, 0) \cup (\frac{2}{3}, \infty)$

CD: $(0, \frac{2}{3})$

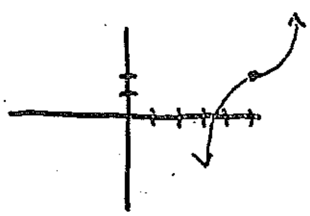
6

4.2-4.6 -1

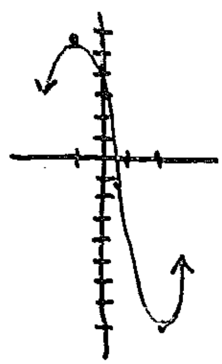
max: $(-1,4)$
min: $(1,0)$
incr: $(-\infty,-1),(1,\infty)$
decr: $(-1,1)$
PI: $(0,2)$
CU: $(0,\infty)$
CD: $(-\infty,0)$



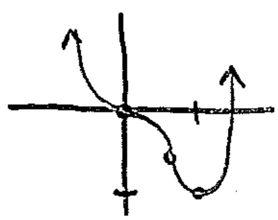
incr: $(-\infty,\infty)$
PI: $(5,2)$
CU: $(5,\infty)$
CD: $(-\infty,5)$



max: $(-1,5.5)$
min: $(2,-8)$
incr: $(-\infty,-1),(2,\infty)$
decr: $(-1,2)$
PI: $(\frac{1}{2},-1.25)$
CU: $(\frac{1}{2},\infty)$
CD: $(-\infty,\frac{1}{2})$



min: $(1,-1)$
incr: $(1,\infty)$
decr: $(-\infty,1)$
PI: $(0,0),(\frac{2}{3},-.59)$
CU: $(-\infty,0),(\frac{2}{3},\infty)$
CD: $(0,\frac{2}{3})$



Name ______________

Calculus 2 Quiz (Practice)
Derivatives

Using the definition of a derivative, find the first derivative of

1. $f(x) = x^2 - 3x$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^2 - 3(x+h) - [x^2+3x]}{h}$$

$$\lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{3x} - 3h - \cancel{x^2} + \cancel{3x}}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(2x+h-3)}{\cancel{h}} \Rightarrow f'(x) \boxed{2x-3}$$

II. Find the first derivative of each of the following. Show all work.

2. $f(x) = 3x^2 - 2x + 3$

$$\boxed{f'(x) = 6x - 2}$$

3. $f(x) = \frac{3}{2x-1}$

$$f'(x) = \frac{(2x-1)0 - 3[2]}{(2x-1)^2}$$

$$\boxed{f'(x) = \frac{-6}{(2x-1)^2}}$$

4. $f(x) = \frac{x^2-3}{x+1}$

$$f'(x) = \frac{(x+1)(2x) - [(x^2-3)(1)]}{(x+1)^2}$$

$$f'(x) = \frac{2x^2 + 2x - x^2 + 3}{(x+1)^2} = \boxed{\frac{x^2+2x+3}{(x+1)^2}}$$

5. $f(x) = (x+3)^3 (2x-1)^4$

$$f'(x) = (x+3)^3 \cdot 4(2x-1)^3(2) + (2x-1)^4 \cdot 3(x+3)^2$$

$$f'(x) = (x+3)^2(2x-1)^3 [8(x+3) + 3(2x-1)]$$

$$(x+3)^2(2x-1)^3 [8x + 24 + 6x - 3]$$

$$f'(x) = (x+3)^2(2x-1)^3 [14x - 21]$$

$$\boxed{7(x+3)^2(2x-1)^3(2x-3)}$$

6. $f(x) = 3 - \frac{4}{x} + \sin x$

$$f(x) = 3 - 4x^{-1} + \sin x$$

$$f'(x) = 4x^{-2} + \cos x$$

$$\boxed{f'(x) = \frac{4}{x^2} + \cos x}$$

7. $f(x) = \frac{3}{\sin x} - \frac{2}{\cos x}$

$$f(x) = 3\csc x - 2\sec x$$

$$\boxed{f'(x) = -3\csc x \cot x - 2\sec x \tan x}$$

8. $f(x) = \sin(6x^2)$

$$f'(x) = \cos(6x^2) \cdot 12x$$

$$\boxed{f'(x) = 12x \cdot \cos(6x^2)}$$

9. $f(x) = \tan^4 x$

$$f(x) = [\tan x]^4$$

$$f'(x) = 4[\tan x]^3 \cdot \sec^2 x$$

$$\boxed{f'(x) = 4\sec^2 x \tan^3 x}$$

10. $f(x) = \cos^3(x^2+1)$

$$f'(x) = 3[\cos(x^2+1)]^2(-\sin(x^2+1))(2x)$$

$$\boxed{f'(x) = -6x \cdot \sin(x^2+1)\cos^2(x^2+1)}$$

11. $f(x) = x \sec x$

$$f'(x) = x\sec x\tan x + \sec x(1)$$

$$\boxed{f'(x) = \sec x[x\tan x + 1]}$$

12. $f(x) = \frac{3}{\sqrt{2x+5}}$

$$f(x) = 3(2x+5)^{-1/2}$$

$$f'(x) = -\frac{3}{2}(2x+5)^{-3/2}(2)$$

$$\boxed{f'(x) = \frac{-3}{(2x+5)^{3/2}}}$$

13. $x^3 + 2xy^2 = 6$ (find $\frac{dy}{dx}$)

$$3x^2 + 2x \cdot 2yy' + y^2(2) = 0$$

$$3x^2 + 4xy \cdot y' + 2y^2 = 0$$

$$\boxed{y' = \frac{-3x^2 - 2y^2}{4xy}}$$

14 Find equations of all horizontal tangents to the graph $y = 2x^3 - 24x - 1$

$y' = 6x^2 - 24$

$y' = 6(x^2 - 4) = 0$

$(x+2)(x-2) = 0$

$x = \pm 2$

$y(2) = 2 \cdot 8 - 24 \cdot 2 - 1$

$= 16 - 48 - 1$

$= -33$

$y + 33 = 0(x-2)$

$(2, -33)$

$\boxed{y = -33}$

$y(-2) = 2(-8) - 24(-2) - 1$

$= -16 + 48 - 1$

$= 31$

$(-2, 31)$

$\boxed{y = 31}$

Calculus II Quiz
Derivative Review

Name: Key

I. **Use the definition of a derivative to find $f'(x)$ for each of the following function. You must show all work and use correct notation.**

1. $f(x) = 2x^2 - 7x$

$$f'(x) = \lim_{h \to 0} \frac{2(x+h)^2 - 7(x+h) - [2x^2 - 7x]}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{7x} - 7h - \cancel{2x^2} + \cancel{7x}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(4x + 2h - 7)}{\cancel{h}}$$

$$\boxed{f'(x) = 4x - 7}$$

2. $f(x) = \sqrt{3x - 5}$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{3(x+h) - 5} - \sqrt{3x-5}}{h} ; \frac{(\sqrt{3x+3h-5} + \sqrt{3x-5})}{(\sqrt{3x+3h-5} + \sqrt{3x-5})}$$

$$\lim_{h \to 0} \frac{\cancel{3x} + 3h \cancel{-5} - (\cancel{3x} \cancel{-5})}{h(\sqrt{\ } + \sqrt{\ })}$$

$$\lim_{h \to 0} \frac{\cancel{h}(3)}{\cancel{h}(\sqrt{\ } + \sqrt{\ })} = \frac{3}{\sqrt{3x-5} + \sqrt{3x-5}}$$

$$= \boxed{\frac{3}{2\sqrt{3x-5}}}$$

II. **Find the derivative for each of the following functions.**

3. $y = \frac{x^6}{5} - \frac{x^4}{3} + 8x + \frac{1}{x^3}$ x^{-3}

$$\boxed{y' = \frac{6}{5}x^5 - \frac{4}{3}x^3 + 8 - \frac{3}{x^4}}$$

4. $y = (x+5)(2x^2 - 1) = 2x^3 + 10x^2 - x - 5$

$$\boxed{y' = 6x^2 + 20x - 1}$$

5. $g(x) = \frac{5x^3 + 3x^2}{x^2} = 5x + 3$

$$\boxed{g'(x) = 5}$$

6. $y = (3x-1)^{-1}(x+4)$ $\frac{x+4}{(3x-1)}$

$$y' = \frac{(3x-1)(1) - (x+4)(3)}{(3x-1)^2}$$

$$= \frac{\cancel{3x} - 1 - \cancel{3x} - 12}{(3x-1)^2}$$

$$\boxed{y' = \frac{-13}{(3x-1)^2}}$$

7. $y = \frac{x-5}{x}$ $= 1 - 5x^{-1}$

$\boxed{y' = \frac{5}{x^2}}$

8. $y = \frac{x}{x-5}$

$y' = \frac{(x-5)(1) - (x)}{(x-5)^2}$

$\boxed{y' = \frac{-5}{(x-5)^2}}$

9. $y = 4x\sqrt{x}$ $= 4x^{3/2}$

$y' = 6x^{1/2}$

$\boxed{y' = 6\sqrt{x}}$

10. Find the equation of the tangent to $f(x) = \frac{1}{4}x^2 - 3$ at $x = -2$.

$m = f'(x) = \frac{1}{2}x$

$f'(-2) = \frac{1}{2}(-2)$

$= -1$

$y = \frac{1}{4} \cdot 4 - 3$

$= -2$

$(-2, -2)$

$y + 2 = -1(x + 2)$

$y + 2 = -x - 2$

$\boxed{y = -x - 4}$

11. Find the equation(s) of all horizontal tangent(s) to the graph of $y = x^3 - 3x - 4$. Show all work.

$\boxed{y = -6}$

$\boxed{y = -2}$

$y(1) = 1 - 3 - 4$

$= -6$

$(1, -6)$

$y(-1) = -1 + 3 - 4$

$= -2$

$(-1, -2)$

$y' = 3x^2 - 3 = 0$

$3(x^2 - 1) = 0$

$x = \pm 1$