CALCULUS 2 CHAIN (NOTES)

Name: Key

WORKSHEET 3.5-1

Find the derivative of each of the following.

1. $y = (5x^2 - 2x + 4)^4$

$y' = 4(5x^2-2x+4)^3(10x-2)$

$y' = 4(10x-2)(5x^2-2x+4)^3$

2. $y = \dfrac{1}{(4x^3+5x^2)^6}$ $\quad y = (4x^3+5x^2)^{-6}$

$y' = -6(12x^2+10x)(4x^3+5x^2)^{-7}$

3. $y = \sqrt{2x^3+3x^2}$ $\quad (2x^3+3x^2)^{1/2}$

$y' = \dfrac{6x^2+6x}{2\sqrt{2x^3+3x^2}}$ $\quad y' = \dfrac{3x(x+1)}{\sqrt{2x^3+3x^2}}$

4. $y = \dfrac{3}{2\sqrt[5]{(3x+7)^4}}$ $\quad y = \frac{3}{2}(3x+7)^{4/5}$

$y' = \frac{3}{2}\cdot\frac{4}{5}(3x+7)^{-1/5}(3)$

$y' = \dfrac{18}{5\sqrt[5]{3x+7}}$

5. $y = \sin 3x + \cos 5x$

$y' = 3\cos(3x) - 5\sin(5x)$

6. $y = 4\sec 5x$

$y' = 4(\sec 5x)\tan(5x)\cdot 5$

$y' = 20\sec(5x)\tan(5x)$

7. $y = \cos(x^3)$

$y' = -3x^2\sin(x^3)$

8. $y = (1+\cos^2 x)^6$

$y' = 6(1+\cos^2 x)^5\cdot(2\cos x)(-\sin x)$

$y' = -10(\sin x)(1+\cos^2 x)^5$

9. $y = \tan(x^2) + \tan^2 x$

$y' = 2x\sec^2(x^2) + 2(\tan x)\sec^2 x$

10. $y = \sin^3 x - \cos^3 x$

$y' = 3(\sin^2 x)\cos x + 3(\cos^2 x)\sin x$

$= 3\sin x\cos x(\sin x + \cos x)$

11. $y = \cot^5(4x)$

$y' = 5[\cot^4(4x)]\cdot(-\csc^2(4x))\cdot 4$

$y' = -20\csc^2(4x)\cot^4(4x)$

12. $y = 3\cos^2(2x)$

$y' = 3[2\cos(2x)(-\sin(2x))\cdot 2]$

$y' = -12\sin(2x)\cos(2x)$

13. $y = \sin(2x+4)^4$

$y' = \cos((2x+4)^4)[4(2x+4)^3(2)]$

$y' = 8(2x+4)^3\cos(2x+4)^4$

14. $y = \sin(\cos(\sin x))$

$y' = \cos(\cos(\sin x))[-\sin(\sin x)]\cos x$

$y' = -\cos x[\sin(\sin x)]\cos(\cos(\sin x))$

15. $y = \cos^2(\cos 4x)$

$y' = 2[\cos(\cos 4x)]'(-\sin(\cos 4x))$

$(-\sin(4x))(4)$

$y' = 8\sin(4x)\cdot\sin(\cos(4x))\cos(\cos(4x))$

16. $y = -[\sec^3(2x^2+x-1)]^3$

$y' = -3[\sec^2(2x^2+x-1)]$

$\sec(2x^2+x-1)\tan(2x^2+x-1)$

$(4x+1)$

17. Find the equation of the tangent to $y = \sqrt[3]{x^2+4}$ at $x = 2$.

$y = \sqrt[3]{8}$ $\quad y = 2$ $\quad (2,2)$

$y' = \frac{1}{3}(x^2+4)^{-2/3}(2x)$

$y'(2) = \dfrac{\frac{1}{3}(4)}{(8)^{2/3}} = \dfrac{\frac{4}{3}(\frac{1}{4})}{4} = \dfrac{1}{3}$

$(y-2) = \frac{1}{3}(x-2)$

$y - 2 = \frac{1}{3}x - \frac{2}{3} + 2$

$y = \frac{1}{3}x + \frac{4}{3}$

18. At which point(s) does the graph of $y = (2x^3 - x^2)^4$ have a horizontal tangent line?

$y' = 4(2x^3-x^2)^3(6x^2-2x) = 0$

$(2x^3-x^2)^3 = 0$ $\quad 6x^2-2x = 0$

$x^2(2x-1) = 0$ $\quad 2x(3x-1) = 0$

$x = 0, \frac{1}{2}$ $\quad x = 0, \frac{1}{3}$