CALCULUS 2

Name: KEY

WORKSHEET 3.3/3.4

→ Product, Quotient, Chain, Trig

Find the derivative of each of the following.

1. $y = (3x^2 + 6)(2x - 1)$
2. $y = (2 - x - 3x^3)(7 + x^5)$
3. $y = (x^3 + 7x^2 - 8)(2x^{-3} + x^{-4})$
4. $y = \left(\frac{1}{x} + \frac{1}{x^2}\right)(3x^3 + 27)$
5. $y = \frac{x^2 + 1}{3x}$
6. $y = \frac{2x - 1}{x + 3}$
7. $y = \frac{1}{5x - 3}$
8. $y = \frac{x^3 - 2}{6}$
9. $y = \frac{4x + 1}{x^2 - 5}$
10. $y = \frac{1}{(2x + 3)(x - 1)}$
11. $y = (x - 2)^{-1}(2x^3 - 1)$
12. $y = (2x^7 - x^2)\left(\frac{x - 1}{x + 1}\right)$
13. $y = \sin x \cos x$
14. $y = x \tan x$
15. $y = \frac{\sin x}{x}$
16. $y = \frac{\sin x}{1 + \cos x}$
17. $y = \sin^2 x$
18. $y = x^2 \cos x$
19. $y = x^3 \sin x - 5 \cos x$
20. $y = (x^2 + 1) \sec x$
21. $y = \sec x \tan x$
22. $y = \csc x \cot x$
23. $y = \frac{\sin x \sec x}{1 + x \tan x}$
24. $y = \frac{\sin x \cos x}{x - x\cos^2 x}$

Find $\frac{d^2y}{dx^2}$ for each of the following.

25. $y = x \cos x$

26. $y = x^2 \sin x - 4 \cos x$

27. Find the equation of the tangent to $y = \frac{x}{\cos x}$ at $x = \pi$

28. At which point(s) does the graph of $y = \frac{x}{x^2 + 9}$ have a horizontal tangent line?

Product, Quotient, Trig

WKSHT: 3.3/3.4

① $y = (3x^2+6)(2x-1)$

$y' = (3x^2+6)(2) + (2x-1)(6x)$

$y' = 6x^2+12+12x^2-6x$

$\boxed{y' = 18x^2-6x+12}$

② $y = (2-x-3x^3)(7+x^5)$

$y' = (2-x-3x^3)(5x^4)+(7+x^5)(-1-9x^2)$

$y' = 10x^4-5x^5-15x^7-7-x^5-63x^2-9x^7$

$\boxed{y' = -24x^7-6x^5+10x^4-63x^2-7}$

④ $y = \left(\frac{1}{x}+\frac{1}{x^2}\right)(3x^3+27)$

$y = 3x^2+27x^{-1}+3x+27x^{-2}$

$y' = 6x-27x^{-2}+3-54x^{-3}$

$\boxed{y' = 6x-\frac{27}{x^2}+3-\frac{54}{x^3}}$

③ $y = (x^3+7x^2-8)(2x^{-3}+x^{-4})$

$y = 2+x^{-1}+14x^{-1}+7x^{-2}-16x^{-3}-8x^{-4}$

$y' = -x^{-2}-14x^{-2}-14x^{-3}+48x^{-4}+32x^{-5}$

$\boxed{y' = \frac{-15}{x^2}-\frac{14}{x^3}+\frac{48}{x^4}+\frac{32}{x^5}}$

⑤ $y = \frac{x^2+1}{3x}$

$y = \frac{1}{3}x+\frac{1}{3}x^{-1}$

$y' = \frac{1}{3}-\frac{1}{3}x^{-2}$

$\boxed{y' = \frac{1}{3}-\frac{1}{3x^2}}$

⑥ $y = \frac{2x-1}{x+3}$

$2x+6-2x+1$

$y' = \frac{(x+3)(2)-[(2x-1)(1)]}{(x+3)^2}$

$\boxed{y' = \frac{7}{(x+3)^2}}$

⑦ $y = \frac{1}{(5x-3)}$

$y = (5x-3)^{-1}$

$y' = -1(5x-3)^{-2}$

$\boxed{y' = \frac{-1}{(5x-3)^2}}$

⑧ $y = \frac{x^3-2}{6}$

$y = \frac{1}{6}x^3-\frac{1}{3}$

$y' = \frac{1}{2}x^2$

$\boxed{y' = \frac{1}{2}x^2}$

(9) $y = \dfrac{4x-1}{x^2-5}$ $\quad 8x^2-2x$

$y' = \dfrac{(x^2-5)(4) - [(4x-1)(2x)]}{(x^2-5)^2}$

$y' = \dfrac{4x^2-20-8x^2+2x}{(x^2-5)^2}$

$\boxed{y' = \dfrac{-4x^2+2x-20}{(x^2-5)^2}}$

(10) $y = \dfrac{1}{(2x+3)(x-1)}$

$y = (2x^2+x-3)^{-1}$

$y' = -1(2x^2+x-3)^{-2}(4x+1)$

$\boxed{y' = \dfrac{-(4x+1)}{(2x^2+x-3)^2}}$

(11) $y = (x-2)^{-1}(2x^3-1)$

$y' = (x-2)^{-1}(6x^2) + (2x^3-1)(-(x-2)^{-2})$

$y' = (x-2)^{-2}[(x-2)(6x^2) - (2x^3-1)]$

$y' = \dfrac{6x^3-12x^2-2x^3+1}{(x-2)^2}$

$\boxed{y' = \dfrac{4x^3-12x^2+1}{(x-2)^2}}$

(12) $y = (2x^7-x^2)\left(\dfrac{x-1}{x+1}\right)$ $\quad (14x^6-2x)$

$y' = (2x^7-x^2)\left[\dfrac{\overset{\cancel{x}+1-\cancel{x}+1}{(x+1)-(x-1)}}{(x+1)^2}\right] + \left[\dfrac{x-1}{x+1}\right]$

$y' = (2x^7-x^2)\left(\dfrac{2}{(x+1)^2}\right)^{(x+1)} + \left(\dfrac{x-1}{x+1}\right)^{(14x^6-2x)(x+2}$

$y' = \dfrac{4x^7-2x^2}{(x+1)^2} + \dfrac{14x^8-2x^3-14x^6+2x}{(x+1)^2}$

$\boxed{y' = \dfrac{14x^8+4x^7-14x^6-2x^3-2x^2+2x}{(x+1)^2}}$

(13) $y = \sin x \cos x$

$y' = \sin x(-\sin x) + \cos x(\cos x)$

$\boxed{y' = -\sin^2 x + \cos^2 x}$

(14) $y = x \tan x$

$y' = x(\sec^2 x) + \tan x(1)$

$\boxed{y' = x\sec^2 x + \tan x}$

(15) $y = \dfrac{\sin x}{x}$

$y' = \dfrac{(x)(\cos x) - \sin x(1)}{x^2}$

$\boxed{y' = \dfrac{x\cos x - \sin x}{x^2}}$

(16) $y = \dfrac{\sin x}{(1+\cos x)}$

$y' = \dfrac{(1+\cos x)(\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}$

$y' = \dfrac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$

$y' = \dfrac{\cancel{\cos x + 1}}{(1+\cos x)^{\cancel{2}}} = \boxed{\dfrac{1}{1+\cos x}}$

(17) $y = \sin^2 x$

$y = (\sin x)^2$

$y' = 2(\sin x)\cos x$

$y' = 2\sin x\cos x$

(18) $y = x^2\cos x$

$y' = x^2(-\sin x) + \cos x(2x)$

$\boxed{y' = -x^2\sin x + 2x\cos x}$

(19) $y = x^3\sin x - 5\cos x$

$y' = x^3(\cos x) + \sin x(3x^2) + 5\sin x$

$\boxed{y' = x^3\cos x + 3x^2\sin x + 5\sin x}$

(20) $y = (x^2+1)(\sec x)$

$y' = (x^2+1)(\sec x\tan x) + \sec x(2x)$

$\boxed{y' = \sec x\left[(x^2+1)\tan x + 2x\right]}$

(21) $y = \sec x\tan x$

$y' = \sec x(\sec^2 x) + \tan x(\sec x\tan x)$

$\boxed{y' = \sec x\left[\sec^2 x + \tan^2 x\right]}$

(22) $y = \csc x\cot x$

$y' = \csc x(-\csc^2 x) + \cot x(-\csc x\cot x)$

$\boxed{y' = -\csc x\left[\csc^2 x + \cot^2 x\right]}$

$$\frac{\sin x}{1}\cdot\frac{1}{\cos x}$$

$$\sin x \sec x$$

(23) $y = 1 + x\tan x$

$$y = \frac{\tan x}{1 + x\tan x}$$

$$y' = \frac{(1+x\tan x)(\sec^2 x) - [\tan x(x\sec^2 x + \tan x(1))]}{(1+x\tan x)^2}$$

$$= \sec^2 x[1 + x\cancel{\tan x} - x\cancel{\tan x} - \tan x]$$

$$\boxed{y' = \frac{\sec^2 x[1 - \tan x]}{(1+x\tan x)^2}}$$

(24) $y = \dfrac{\sin x\cos x}{x(1-\cos^2 x)}$

$y = \dfrac{\cancel{\sin x}\cos x}{x\cdot\sin^2 x}$ $\quad y = \dfrac{\cos x}{x\sin x}$ $\quad y = \frac{1}{x}\cot x$

$y = x^{-1}\cot x$

$$y' = x^{-1}\cot x$$

$$y' = x^{-1}(-\csc^2 x) + \cot x(-x^{-2})$$

$$= -x^{-2}(x\csc^2 x + \cot x)$$

$$\boxed{y' = \frac{-(x\csc^2 x + \cot x)}{x^2}}$$

(25) $y = x\cos x$

$$y' = x(-\sin x) + (\cos x)$$

$$y' = -x\sin x + \cos x$$

$$y'' = -x(\cos x) + \sin x(-1) + (-\sin x)$$

$$\boxed{y'' = -x\cos x - 2\sin x}$$

(26) $y = x^2 \sin x - 4\cos x$

$y' = x^2(\cos x) + \cos x(2x) + 4\sin x$

$y' = x^2\cos x + 2x\cos x + 4\sin x$

$y'' = x^2(-\sin x) + \cos x(2x) + 2x(-\sin x) + \cos x(2) + 4\cos x$

$\boxed{y'' = -x^2\sin x + 2x\cos x - 2x\sin x + 2\cos x + 4\cos x}$

(27) $y = \dfrac{x}{\cos x}$ @ $x = \pi$ $(\pi, -\pi)$

$y(\pi) = \dfrac{\pi}{\cos(\pi)}$ $y = -\pi$

-1 0 $y(\pi) = \dfrac{\pi}{-1} = -\pi$

$y + \pi = m(x - \pi)$

$y + \pi = -1(x - \pi)$

$y + \pi = -x + \pi$

$\boxed{y = -x}$

$y' = \dfrac{(\cos x)(1) - x(-\sin x)}{(\cos x)^2}$

$y'(\pi) = \dfrac{\cos\pi + \pi\sin(\pi)}{(\cos\pi)^2} = \dfrac{-1+0}{(-1)^2} = -1$

(28) $y = \dfrac{x}{x^2+9}$

$y' = \dfrac{(x^2+9)(1) - [x(2x)]}{(x^2+9)^2} = 0$

horizontal
just numer$=0$

$y' = x^2 + 9 - 2x^2 = 0$

$-x^2 + 9 = 0$

$x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$\boxed{x = 3, -3}$

so when $x=3$ $y = \frac{3}{9+9} = \frac{3}{18} = \frac{1}{6}$

$(3, 1/6)$

when $x = -3$ $y = \frac{-3}{9+9} = \frac{-1}{6}$

$(-3, -1/6)$