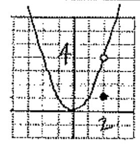
Central Bucks School District
Calculus 1 - HS Review for Final Exam

1) Find the $\lim_{x\to 2} f(x)$ if $f(x) = \begin{cases} x^2, & x \neq 2 \\ 1, & x = 2 \end{cases}$



$\lim_{x\to 2^+} 4$ $\lim_{x\to 2^-} 4$ $= \boxed{4}$

2) Find $\lim_{x\to 2}\left(4x^3 - 5x\right) = (4\cdot 2^3 - 5\cdot 2)$
$(32-10)$ $= \boxed{22}$

3) Find the point(s) on the graph of the function $f(x) = 3x^4 - 3x^2 + 2x$ where the slope of the tangent is 2.

$f'(x) = 12x^3 - 6x + 2$
$f'(x) = 2(6x^3 - 3x + 1) = 2$
$6x^3 - 3x + 1 = 1$
$6x^3 - 3x = 0$
$3x(2x^2 - 1) = 0$
$x = 0 \quad x^2 = \frac{1}{2}$
$\boxed{x = 0 \quad x = \pm\sqrt{1/2}}$

4) At which values of x is $f(x) = \left[(x-4)(x-7)\right]/(x-4)$ discontinuous?
$\boxed{x = 4}$

5) Find $\lim_{x\to 0^+}\left(\frac{1}{x^2}\right)$. $\lim_{x\to 0^+}\frac{1}{x^2}$ $\boxed{+\infty}$

6) Find all vertical asymptotes of $g(x) = \frac{(x+5)}{x^2 - 25}$. $\boxed{x = 5}$
$(x+5)(x-5)$

7) If $f(x) = x^2 - 2$, what is the derivative of $f(x)$ using the definition of the derivative?
$f'(x) = 2x$

$\lim_{h\to 0}\frac{((x+h)^2 - 2) - (x^2 - 2)}{h}$
$\lim_{h\to 0}\frac{x^2 + 2xh + h^2 - 2 - x^2 + 2}{h}$
$\lim_{h\to 0}\frac{h(2x+h)}{h}$ $\boxed{2x}$

8) Find $f'(x)$ if $f(x) = \frac{2x-3}{\cos x}$.
$\frac{\cos x\,(2) + (2x-3)\sin x}{\cos^2 x}$
$= \boxed{\frac{2\cos x + \sin x\,(2x-3)}{\cos^2 x}}$

9) Find the slope of the tangent to $f(x) = \left(\sqrt{7}\,\sin x\right)$, at $x = \pi/4$.
$f'(x) = \sqrt{7}\cos x$ $\quad f'(\pi/4) = \sqrt{7}\cos\left(\frac{\pi}{4}\right) = \sqrt{7}\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\sqrt{14}}{2}}$
≈ 1.87

10) Find an equation for the tangent line to the graph of $f(x) = 5x^3 + 2x^2 - 4$ at the point where $x = 1$.
$y - 3 = m(x-1)$
$y - 3 = 19(x-1)$
$y - 3 = 19x - 19$
$\boxed{y = 19x - 16}$
$f(1) = 5 + 2 - 4$
$y = 3$
$f'(x) = 15x^2 + 4x$
$f'(1) = 15 + 4 = 19$

11) Find all points on the graph of $f(x) = -x^4 + 2x^3 - 2$ at which there is a horizontal tangent line.
$f'(x) = -4x^3 + 6x^2$ $\quad f'(x) = -2x^2(2x+3) = 0$
$x \neq (0, -2)$ $\quad x = (-3/2, -\frac{5}{16})$

12) Find $f'(x)$ for $f(x) = (x^2)(\cos x)$.
$f'(x) = -x^2\sin x + \cos x \cdot 2x$
$= \boxed{-x^2\sin x + 2x\cos x}$

13) Find $f'(x)$ for $f(x) = \left(3x^5 - 7x\right)^4$
$f'(x) = 4(3x^5 - 7x)^3 \cdot (15x^4 - 7)$
$= \boxed{4(15x^4 - 7)(3x^5 - 7x)^3}$

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Calculus 1 - HS Review for Final Exam

14) Find the instantaneous rate of change of k with respect to b if $k = 2b^3 + 4b^2 - 18$.

$\boxed{K' = 6b^2 + 8b}$

15) A particle moves along the curve given by $y = -x^4 - 4$. Find the acceleration at 7 seconds.

$y' = -4x^3$

$y''(7) = -12x^2 \Rightarrow -12(49) \Rightarrow$ accel $= -588$

16) Find y' if $x - y^3 - 6xy = 2$.

$1 - 3y^2 \cdot y' - (6xy' + y(6)) = 0$

$+3y^2 \cdot y' + 6xy' = +1 - 6y$

$y'(3y^2 + 6x) = 1 - 6y$ ✓ $\boxed{y' = \dfrac{1-6y}{3y^2+6x}}$

17) A balloon is released at a point 500 yards away from an observer who is on level ground. If the balloon goes straight up at a rate of 15 yards every second, how fast is the distance from the observer to the balloon increasing when the balloon is 200 yards high?

$\frac{dr}{dt} =$; $r = 538.5$; $\frac{dy}{dt} = 15$ y/s ; $y = 200$; $x = 500$; $\frac{dx}{dt} =$

$(500)^2 + (200)^2 = r^2$

$250{,}000 + 40000 = r^2$ $\quad r = 538.5$

$x\frac{dx}{dt} + y\frac{dy}{dt} = r\frac{dr}{dt}$

$(500)(0) + (200)(15) = 538.5\frac{dr}{dt}$

$\boxed{5.57 \text{ yd/sec} = \frac{dr}{dt}}$

18) The position function for an object is given by $s(t) = t^5 + \frac{4}{3}t^2 + 5$, where s is measured in feet and t is measured in seconds. What is the instantaneous velocity when t = 1.3 seconds?

$s'(1.3) = 5t^4 + 8/3\,t \Rightarrow s'(1.3) = 14.28 + 3.47$

$= \boxed{17.7 \text{ ft/sec}}$

19) Where is the function, $f(x) = \frac{1}{3}x^6 - 9x^2 - 5$, increasing?

$f'(x) = 2x^5 - 18x$

$= 2x(x^4 - 9)$

$= 2x(x^2-3)(x^2+3)$

$x = \pm\sqrt{3}$

$-\ 0\ +\ 0\ -\ 0\ +$ at $-\sqrt{3}$ (min), 0 (max), $\sqrt{3}$ (min)

$\boxed{[-\sqrt{3}, 0] \cup [\sqrt{3}, \infty)}$

20) Find the derivative of $f(x) = 12x^{10} - 12x^5 + 3x^3 - 5x^2 - 3$

$\boxed{f'(x) = 120x^9 - 60x^4 + 9x^2 - 10x}$

21) Find all values of x that give relative extrema for the function $f(x) = 2x^4 - 7x^2$.

$f'(x) = 8x^3 - 14x = 0$

$2x(4x^2 - 7) = 0$

$x = 0 \quad x = \pm\sqrt{\frac{7}{4}}$ or $\pm\frac{\sqrt{7}}{2}$

$0\ \ 0\ -\ 0$ at $-\frac{\sqrt{7}}{2}$ (min), 0 (max), $\frac{\sqrt{7}}{2}$ (min)

22) Find the interval on which the function $f(x) = -x^3 - 4x^2 + 28x + 48$ is concave up.

$f'(x) = -3x^2 - 8x + 28 =$

$f''(x) = -6x - 8 = 0$

$x = \frac{8}{-6} = \frac{4}{3}$

$+\ 0\ -$ at $-\frac{4}{3}$

$\boxed{\text{ccu } (-\infty, -4/3)}$

23) Find $\lim_{x\to\infty} \dfrac{6x^4 - 8x^3 - 5x}{5x^4 + 5} = \boxed{\dfrac{6}{5}}$

24) Find $f'(x)$ given $f(x) = \cos^2(3x^2)$

$f(x) = [\cos(3x^2)]^2$

$f'(x) = 2[\cos(3x^2)]^1 \cdot -\sin(3x^2) \cdot 6x$

$\boxed{-12x \cdot \sin(3x^2) \cdot \cos(3x^2)}$

25) Integrate $\int \cos^5 x \sin x\, dx$.

$u = \cos x$, $du = -\sin x$

$-\int u^5 du \quad -\frac{u^6}{6} \quad \boxed{-\frac{(\cos x)^6}{6} + C}$

26) Integrate $\int -6x\sqrt{(2x^2 - 5)}\,dx$

$u = 2x^2 - 5$

$du = 4x\,dx$

$\frac{1}{4} \cdot -6\int u^{1/2} du$

$-\frac{3}{2} \cdot \frac{2}{3} u^{3/2}$

$\boxed{-(2x^2-5)^{3/2} + C}$

Calculus 1 - HS Review for Final Exam

27) Find the position function S at time t given the acceleration is 32, the velocity is 20 and the position is 5 when t = 0.

$a(t)=32$
$v(t)=32t+20$
$\boxed{S(t)=16t^2+20t+5}$

28) Integrate $\int \frac{1}{x^2}\,dx$ from x = 1.5 to x = 3.3.

$\int_{1.5}^{3.3} x^{-2}dx \Rightarrow -x^{-1}\Big| \Rightarrow -\frac{1}{x}\Big|_{1.5}^{3.3}$ $\boxed{.3636}$

29) Integrate $\int \sqrt[5]{(x^4)}\,dx$

$\int x^{4/5}dx \Rightarrow \boxed{\frac{5}{9}x^{9/5}+C}$

30) Integrate $\int (5x^5 - 7x^3 + 4)\,dx$.

$\boxed{\frac{5}{6}x^6 - \frac{7}{4}x^4 + 4x + C}$

31) Find the average value of f(x) = $(-4x^5 + 7)$ on the interval $[-1, 3]$.

$\frac{1}{3+1}\int_{-1}^{3} -4x^5+7$
$\frac{1}{4}\left[-\frac{2x^6}{3} + 7x\right]_{-1}^{3}$
$\boxed{-\frac{343}{3} \text{ or } -114.3}$

32) Functions f and g and their derivatives have the following values when $x = 5$;

$f(5) = 7$, $f'(5) = 0.5$, $g(5) = -9$ and $g'(5) = \frac{-1}{3}$. Find $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$ at x=5.

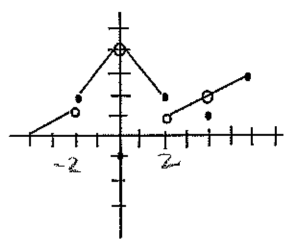
$\frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

$= \frac{-9(\frac{1}{2}) - (7)(-\frac{1}{3})}{(-9)^2} = \frac{-\frac{9}{2} + \frac{7}{3}}{81}$

$\boxed{-\frac{13}{486}}$

$\approx -.0267$

33) Find $\lim_{x \to 2^+} f(x)$.



34) Is f(x) continuous at x = -2?

discont b/c jump

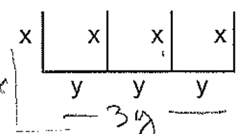
35) Find all the point(s) of inflection for f(x) = $12x^3 - 6x + 2$.

$f'(x) = 36x^2 - 6$
$f''(x) = 72x = 0$

$-\ \ 0\ \ +$

$\boxed{x=0 \quad y=2}$
$(0,2)$

36) A rectangular plot of ground is to be fenced. One side is to remain open and the plot will be divided into 3 equal stalls. If there is 1000 feet of fence, find the value of y that will maximize the area.



$A = lw$
$A = x(3y)$
$A = x(1{,}000 - 4x)$
$A = 1{,}000x - 4x^2$
$A' = 1{,}000 - 8x = 0$
$8x = 1{,}000$
$\boxed{x = 125 \quad y = \frac{500}{3} \text{ or } 166.67}$

Fence =
$1{,}000 = 4x + 3y$
$1{,}000 - 4x = 3y$
$\frac{1{,}000 - 4(125)}{3} = y$

37) Integrate $\int (x)(\sin x^2)\,dx$

$\frac{1}{2}\int \sin u\,du \quad u = x^2$
$du = 2x\,dx$
$\boxed{-\frac{1}{2}\cos x^2 + C}$

38) Integrate $\int \frac{(x^5 - 3x^4 + 2)}{(x^3)}\,dx$

$\int x^2 - 3x + 2x^{-3}dx$
$\boxed{\frac{x^3}{3} - \frac{3}{2}x^2 - x^{-2} + C}$
or $\frac{1}{3}x^3 - \frac{3}{2}x^2 - \frac{1}{x^2} + C$

39) Find f (x) if f "(x) = x + 2, f'(2) = 4 and f(0) = 8.

$f'(x) = \frac{x^2}{2} + 2x + C$
$4 = \frac{4}{2} + 2(2) + C$
$-2 = C$

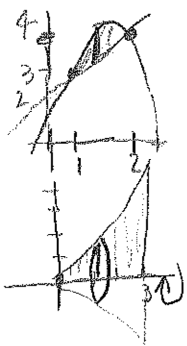
$f'(x) = \frac{1}{2}x^2 + 2x - 2$
$f(x) = \frac{1}{6}x^3 + x^2 - 2x + C$
$8 = 0 + 0 - 0 + C$
$8 = C$
$\boxed{f(x) = \frac{1}{6}x^3 + x^2 - 2x + 8}$

40) Define f(-2) such that it makes f(x) = $\frac{(x^2 - 4x - 12)}{(x+2)}$ continuous at x = -2.

$\frac{(x-6)\cancel{(x+2)}}{\cancel{(x+2)}}$

$f(x) = x - 6$
$f(-2) = -2 - 6$
$= \boxed{-8}$

Calculus 1 - HS Review for Final Exam



41) Integrate $\int \left(\sqrt[3]{x} + \frac{5}{4x^4} + 2x \right) dx.$ $\int x^{1/3} + \frac{5}{4} x^{-4} + 2x \, dx$

$\frac{3}{4} x^{4/3} - \frac{5}{12} x^{-3} + x^2 + C \Rightarrow \boxed{\frac{3}{4} x^{4/3} - \frac{5}{12x^3} + x^2 + C}$

42) Determine the area of the region bounded by $y = -x^2 + 4x$ and $y = x + 2$. $\int_1^2 (-x^2+4x) - (x+2) dx$

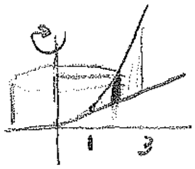
$\int_1^2 -x^2 + 3x - 2 dx = \boxed{\frac{1}{6}} = .167$

43) Find the volume of the solid generated when the area between the $y = x^{5/2}$ and the x-axis from $x = 0$ to $x = 3$ is revolved around the x-axis. $V = \pi \int_0^3 (x^{5/2})^2 dx$

$= \pi \int_0^3 x^5 dx \Rightarrow \frac{243}{2}\pi \approx 121.5\pi$

44) Set up but do not integrate the integral to find the volume of the solid generated by revolving the area in the first quadrant bounded by $y = 3x - x^3$ and the x-axis about the y-axis.

Shell: $2\pi \int_0^{1.73} (x)(3x - x^3) \, dx$



45) What integral expresses the volume of the solid generated by revolving the region bounded by the graph of $y = x^4$ and the line $y = x$, between $x = 1$ and $x = 3$, about the y-axis.

Shell: $\pi \int_1^3 (x)(x^4 - x) dx$

Top - Bottom

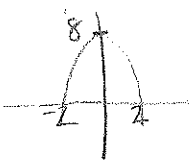
46) If $\int f(x) \, dx = 5x^4 - 3x^2 + 5x + 2$, find $f(x)$.

$f(x) = 20x^3 - 6x + 5$

47) $\int_1^7 g(x) \, dx = -6$, $\int_5^7 g(x) \, dx = 5$, find $\int_1^5 g(x) \, dx$. $\int_1^5 x + \int_5^7 5 = \int_1^7 -6$

$x + 5 = -6$

$x = -11$



48) Determine the area of the region bounded by $y = 0$, $y = -2x^2 + 8$.

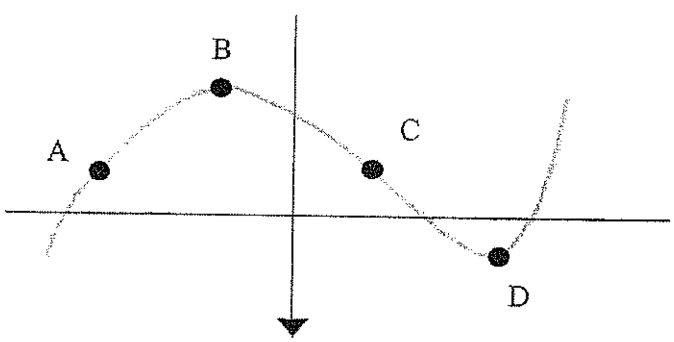
$A = \int_{-2}^{2} (-2x^2 + 8) dx = 64/3 \approx 21.3$

49) Determine the area of the region bounded by $x = y^2 + 2$, $y = 1$, $y = 2$, and $x = 0$.

$A = \int_1^2 (y^2 + 2) dy = 13/3 \approx 4.\overline{3}$

Right - Left

50) Determine the slope of a tangent at the indicated points as positive, negative, zero, or no slope.



A Positive

B $m = 0$

C Negative

D $m = 0$

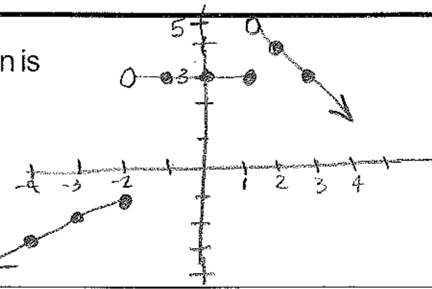
Calculus 1 - HS Review for Final Exam

51. Sketch the piecewise function whose equation is

x	y = x+1
-2	-1
-3	-2
-4	-3

x	y = -x+6
1	5
2	4
3	3

$$f(x)=\begin{cases} x+1; & x\le -2 \\ 3; & -2<x\le 1 \\ -x+6; & x>1 \end{cases}$$



52. Find y' if $x^3+y^2-3xy=4$

$3x^2+2y\cdot y'-(3xy'+y(3))=0$

$3x^2+2y\cdot y'-3xy'-3y=0$

$y'(2y-3x)=3y-3x^2$

$\boxed{y'=\dfrac{3y-3x^2}{2y-3x}}$ or $\dfrac{3x^2-3y}{3x-2y}$

53. If the velocity of a particle is v (t) = 2t - 3, and the initial positions is s = 10, when $t=0$, find the position function.

$s(t)=t^2-3t+C$

$10=0-0+C$

$10=C$

$\boxed{s(t)=t^2-3t+10}$

54. Find $\int_1^3 (3x^2+2)\,dx$

$x^3+2x\Big|_1^3$

$(27+6)-(1+2(1))$

$33-3=\boxed{30}$

55. Find $\int_0^{\sqrt{3}} x\sqrt{x^2+1}\,dx$

$u=x^2+1$

$du=2x\,dx$

$\frac{1}{2}\int_0^{\sqrt{3}} u^{1/2}dx$

$\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}u^{3/2}$

$\frac{1}{3}(x^2+1)^{3/2}\Big|_0^{\sqrt{3}}$

$\frac{1}{3}\left[(3+1)^{3/2}-(0+1)^{3/2}\right]$

$\frac{1}{3}(8-1)=\boxed{\dfrac{7}{3}}$

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Answer Key

1) 4

2) 22

3) $0, \frac{\pm\sqrt{2}}{2}$ or $0, \pm\sqrt{\frac{1}{2}}$

4) 4

5) ∞

6) x= 5

7) 2x

8) $f'(x) = \frac{2\cos x + (2x-3)\sin x}{\cos^2 x}$

9) $\frac{\sqrt{14}}{2} \approx 1.87$

10) y = 19x - 16

11) (0,-2); $\left(\frac{3}{2}, \frac{-5}{16}\right)$

12) 2xcosx - $x^2 \sin x$

13) $4\left(15x^4 - 7\right)\left(3x^5 - 7x\right)^3$

14) $6b^2 + 8b$

15) -588

16) $\frac{1-6y}{6x+3y^2}$

17) $\frac{30}{\sqrt{29}} \approx 5.57 \frac{\text{yds.}}{\text{sec.}}$

18) 17.7 $\frac{ft.}{\text{sec.}}$

19) $\left[\sqrt{3}, 0\right]\left[\sqrt{3}, \infty\right]$

20) $120x^9 - 60x^4 + 9x^2 - 10x$

21) Max $x = 0$

Min. $x = \pm\frac{\sqrt{7}}{2}$

22) $\left(-\infty, -\frac{4}{3}\right)$

23) $\frac{6}{5}$

24) $f'(x) = -12x\cos(3x^2)\sin(3x^2)$

25) $-\frac{1}{6}\cos^6 x + C$

26) $-\left(2x^2 - 5\right)^{\frac{3}{2}} + C$

27) $S = 16t^2 + 20t + 5$

28) .364

29) $\frac{5}{9} x^{\frac{9}{5}} + C$

30) $\frac{5}{6}x^6 - \frac{7}{4}x^4 + 4x + C$

31) $\frac{-343}{3}$

32) $\frac{-13}{486} \approx -.0267$

33) 1

34) No

35) (0, 2)

36) $\frac{500}{3} = 166\frac{2}{3}$ ft.

37) $-\frac{1}{2}\cos\left(x^2\right) + C$

38) $\frac{x^3}{3} - \frac{3}{2}x^2 - \frac{1}{x^2} + C$

39) $f(x) = \frac{1}{6}x^3 + x^2 - 2x + 8$

40) f(-2) = -8

41) $\frac{3}{4}x^{\frac{4}{3}} - \frac{5}{12x^3} + x^2 + C$

42) $\frac{1}{6}$

43) $\frac{243\pi}{2}$

44) $2\pi\int_0^{\sqrt{3}} \left(3x^2 - x^4\right)\, dx$

45) $2\pi\int_1^3 \left(x^5 - x^2\right) dx$

46) $20x^3 - 6x + 5$

47) -11

48) 64/3

49) 13/3

50) A positive
B 0
C negative
D 0