AP Calculus BC
Review – FTC and Separable Differential Equations FRQ

1. 2009 BC4

Consider the differential equation $\frac{dy}{dx} = 6x^2 - x^2y$. Let $y = f(x)$ be a particular solution to this differential equation with the initial condition $f(-1) = 2$.

Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

2. 2007 BC4

Let f be the function defined for $x > 0$, with $f(e) = 2$ and f', the first derivative of f, given by $f'(x) = x^2 \ln x$.

(a) Write an equation for the line tangent to the graph of f at the point $(e, 2)$.

(b) Is the graph of f concave up or concave down on the interval $1 < x < 3$? Give a reason for your answer.

(c) Use antidifferentiation to find $f(x)$.

3. 2006B BC5

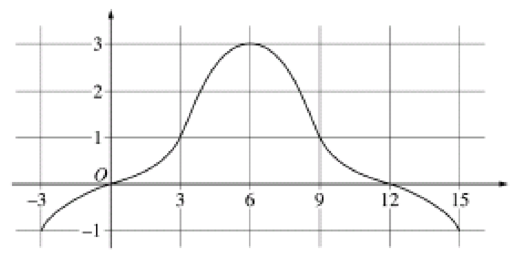
Let f be a function with $f(4) = 1$ such that all points (x, y) on the graph of f satisfy the differential equation

$$\frac{dy}{dx} = 2y(3 - x).$$

Find $y = f(x)$.

4. 2002B BC4

The graph of a differentiable function f on the closed interval $[-3,15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_6^x f(t)\,dt$ for $-3 \le x \le 15$.



Graph of f

(a) Find $g(6)$, $g'(6)$, and $g''(6)$.

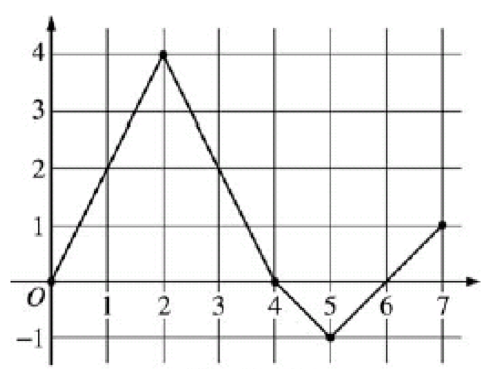
(b) On what intervals is g decreasing? Justify your answer.

(c) On what intervals is the graph of g concave down? Justify your answer.

(d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\Delta t = 3$.

5. 2003B BC5

Let f be a function defined on the closed interval $[0, 7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$.



Graph of f

(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

(b) Find the average rate of change of g on the interval $0 \le x \le 3$.

(c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Explain your reasoning.

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

AP Calculus BC
Review – FTC and Separable Differential Equations FRQ

1. 2009 BC4

$x^2(6-y)$

Consider the differential equation $\frac{dy}{dx} = 6x^2 - x^2y$. Let $y = f(x)$ be a particular solution to this differential equation with the initial condition $f(-1) = 2$.

Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

$\frac{1}{6-y}\,dy = x^2\,dx$

$-\ln|6-y| = \frac{1}{3}x^3 + C$

$\ln|6-y| = -\frac{1}{3}x^3 + C$

$6-y = Ce^{-1/3x^3}$

$-y = Ce^{-1/3x^3} - 6$

$y = Ce^{-1/3x^3} + 6$

$2 = Ce^{1/3} + 6$

$-4 = Ce^{1/3}$

$\frac{-4}{e^{1/3}} = C$

$y = \frac{-4}{e^{1/3}}e^{-1/3x^3}$

$= \boxed{-4e^{-1/3x^3 - 1/3}}$

2. 2007 BC4

Let f be the function defined for $x > 0$, with $f(e) = 2$ and f', the first derivative of f, given by $f'(x) = x^2 \ln x$.

(a) Write an equation for the line tangent to the graph of f at the point $(e, 2)$.

(b) Is the graph of f concave up or concave down on the interval $1 < x < 3$? Give a reason for your answer.

(c) Use antidifferentiation to find $f(x)$.

a) $f'(e) = e^2$

$\boxed{y-2 = e^2(x-e)}$

b) $f'' = x^2 \cdot \frac{1}{x} + \ln x \cdot 2x$

$= x + 2x\ln x = 0$

$= x(1+2\ln x) = 0$

$2\ln x = -1$

$\ln x = -1/2$

$x = e^{-1/2}$

$++ \quad - \quad ++$; 0, $e^{-1/2}$

f is concave down on $(0, e^{-1/2})$ since $f'' < 0$.

c) $\int x^2 \ln x\,dx$ $\quad u = \ln x \quad dv = x^2$ $\quad du = 1/x \quad v = \frac{1}{3}x^3$

$\frac{1}{3}x^3\ln x - \frac{1}{3}\int x^2\,dx$

$\boxed{\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C}$

3. 2006B BC5

Let f be a function with $f(4) = 1$ such that all points (x, y) on the graph of f satisfy the differential equation

$$\frac{dy}{dx} = 2y(3-x).$$

Find $y = f(x)$.

$\frac{1}{y}\,dy = 2(3-x)\,dx$

$\ln|y| = 2(3x - \frac{1}{2}x^2) + C$

$\ln|y| = 6x - x^2 + C$

$y = Ce^{6x-x^2}$

$1 = Ce^{6(4)-4^2}$

$1 = Ce^{8}$

$e^{-8} = C$

$y = e^{-8} \cdot e^{6x-x^2}$

$\boxed{y = e^{6x-x^2-8}}$

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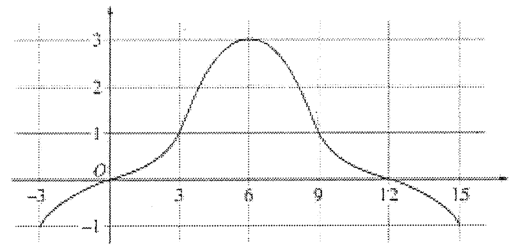

Graph of f

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(c) On what intervals is the graph of g concave down? Justify your answer.

(d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\Delta t = 3$.

a) $g(6) = 5 + \int_6^6 f(t)dt = 5$

$g'(6) = f(6) = 3$

$g''(6) = f'(6) = 0$

b) $g' = f < 0$ on $(-\infty, 0) \cup (12, \infty)$

c) $g'' = f' < 0$ on $(6, \infty)$

d) $\frac{1}{2}(3)(-1 + 2(0) + 2(1) + 2(3) + 2(1) + 2(0) + -1)$

$= \frac{3}{2}(8)$

$= 12$

5. 2003B BC5

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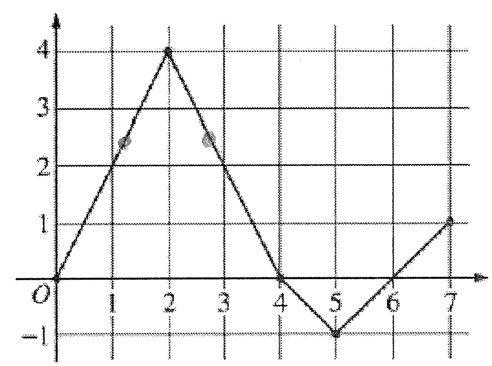

Graph of f

(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

(b) Find the average rate of change of g on the interval $0 \le x \le 3$.

(c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Explain your reasoning.

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

a) $g(3) = \int_2^3 f(t)dt = \frac{1}{2}(6) = 3$

$g'(3) = f(3) = 2$

$g''(3) = f'(3) = -2$

b) $\frac{g(3) - g(0)}{3 - 0} = \frac{3 - (-4)}{3} = \frac{7}{3}$

c) $g'(c) = f(c) = 7/3$ AT TWO VALUES.

d) $g'' = f'$ CHANGES SIGN AT $x = 2, 5$.