AP Calculus BC
Chapter 6 - AP Exam Problems

Antiderivatives

1. $\int \left(x^2+1\right)^2 dx =$

A) $\frac{\left(x^2+1\right)^3}{3}+C$ C) $\left(\frac{x^3}{3}+x\right)^2+C$ E) $\frac{x^5}{5}+\frac{2x^3}{3}+x+C$

B) $\frac{\left(x^2+1\right)^3}{6x}+C$ D) $\frac{2x\left(x^2+1\right)^3}{3}+C$

2. If the second derivative of *f* is given by $f''(x)=2x-\cos x$, which of the following could be $f(x)$?

A) $\frac{x^3}{3}+\cos x-x+1$ C) $x^3+\cos x-x+1$ E) $x^2+\sin x+1$

B) $\frac{x^3}{3}-\cos x-x+1$ D) $x^2-\sin x+1$

3. Find $\int \sec^2 x\,dx =$

A) $\tan x+C$ C) $\cos^2 x+C$ E) $2\sec^2 x\tan x+C$

B) $\csc^2 x+C$ D) $\frac{\sec^3 x}{3}+C$

Evaluating Definite Integrals

4. If *f* is a linear function and 0 < *a* < *b*, then $\int_a^b f''(x)dx =$

A) 0 B) 1 C) $\frac{ab}{2}$ D) $b-a$ E) $\frac{b^2-a^2}{2}$

5. Find $\int_1^2 x^{-3}dx =$

A) $-\frac{7}{8}$ B) $-\frac{3}{4}$ C) $\frac{15}{64}$ D) $\frac{3}{8}$ E) $\frac{15}{16}$

6. If $\int_{-2}^{2}\left(x^7+k\right)dx=16$, then k =

A) –12 B) –4 C) 0 D) 4 E) 12

7. Find $\int_1^2 \frac{1}{x^2}dx =$

A) $-\frac{1}{2}$ B) $\frac{7}{24}$ C) $\frac{1}{2}$ D) 1 E) $2\ln 2$

8. Find $\int_1^e \left(\frac{x^2-1}{x}\right)dx =$

A) $e-\frac{1}{e}$
B) e^2-e
C) $e^2-e+\frac{1}{2}$
D) e^2-2
E) $\frac{e^2}{2}-\frac{3}{2}$

9. Find $\int_0^1 (3x-2)^2\,dx =$

A) $-\frac{7}{3}$ B) $-\frac{7}{9}$ C) $\frac{1}{9}$ D) 1 E) 3

10. Find $\int_{-1}^1 \frac{3}{x^2}dx =$

A) -6 B) -3 C) 0 D) 6 E) nonexistent

11. Find $\int_1^2 \frac{x^2-1}{x+1}dx =$

A) $\frac{1}{2}$ B) 1 C) 2 D) $\frac{5}{2}$ E) $\ln 3$

12. If $\int_0^k \left(2kx-x^2\right)dx = 18$, then $k =$

A) -9 B) -3 C) 3 D) 9 E) 18

13. What are all values of k for which $\int_{-3}^k x^2dx = 0$?

A) -3 B) 0 C) 3 D) -3 and 3 E) -3, 0, and 3

14. Which of the following is equal to $\int_0^{\pi} \sin x\, dx$?

A) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\, dx$ C) $\int_{-\pi}^{0} \sin x\, dx$ E) $\int_{0\pi}^{2\pi} \sin x\, dx$

B) $\int_0^{\pi} \cos x\, dx$ D) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x\, dx$

15. Find $\int_0^3 |x-1|\, dx =$

A) 0 B) $\frac{3}{2}$ C) 2 D) $\frac{5}{2}$ E) 6

16. Find $\int_1^4 |x-3|\, dx =$

A) $-\frac{3}{2}$ B) $\frac{3}{2}$ C) $\frac{5}{2}$ D) $\frac{9}{2}$ E) 5

17. If the function f has a continuous derivative on $[0, c]$, then $\int_0^c f'(x)\, dx =$

A) $f(c)-f(0)$ C) $f(c)$ E) $f''(c)-f''(0)$
B) $|f(c)-f(0)|$ D) $f(x)+C$

18. If $f(x)=\begin{Bmatrix} x & \text{for } x \le 1 \\ \frac{1}{x} & \text{for } x > 1 \end{Bmatrix}$, then $\int_0^e f(x)\, dx =$

A) 0 B) $\frac{3}{2}$ C) 2 D) e E) $e+\frac{1}{2}$

19. Find $\int_1^{500} \left(13^x - 11^x\right)dx + \int_2^{500} \left(11^x - 13^x\right)dx =$

A) 0.000 B) 14.946 C) 34.415 D) 46.000 E) 136.364

20. If p is a polynomial of degree n, $n > 0$, what is the degree of the polynomial $Q(x)=\int_0^x p(t)dt$?

A) 0 B) 1 C) $n-1$ D) n E) $n+1$

21. (1988 AB6) Let f be the differentiable function, defined for all real numbers x, with the following properties.

(i) $f'(x) = ax^2 + bx$
(ii) $f'(1) = 6$ and $f''(1) = 18$
(iii) $\int_1^2 f(x)dx = 18$

Find $f(x)$. Show your work.

U-Substitutions

22. Find $\int \frac{x\,dx}{\sqrt{3x^2+5}} =$

A) $\frac{1}{9}(3x^2+5)^{\frac{3}{2}}+C$
B) $\frac{1}{4}(3x^2+5)^{\frac{3}{2}}+C$
C) $\frac{1}{12}(3x^2+5)^{\frac{1}{2}}+C$
D) $\frac{1}{3}(3x^2+5)^{\frac{1}{2}}+C$
E) $\frac{3}{2}(3x^2+5)^{\frac{1}{2}}+C$

23. Find $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{1+\sin\theta}}d\theta =$

A) $-2(\sqrt{2}-1)$
B) $-2\sqrt{2}$
C) $2\sqrt{2}$
D) $2(\sqrt{2}-1)$
E) $2(\sqrt{2}+1)$

24. Find $\int \frac{3x^2}{\sqrt{x^3+1}}dx =$

A) $2\sqrt{x^3+1}+C$
B) $\frac{3}{2}\sqrt{x^3+1}+C$
C) $\sqrt{x^3+1}+C$
D) $\ln(x^3+1)+C$
E) $\ln\sqrt{x^3+1}+C$

25. Find $\int_0^1 x(x^2+2)^2 dx =$

A) $\frac{19}{2}$ B) $\frac{19}{3}$ C) $\frac{9}{2}$ D) $\frac{19}{6}$ E) $\frac{1}{6}$

26. If $\frac{dy}{dx} = \cos(2x)$, then $y =$

A) $-\frac{1}{2}\cos(2x) + C$
B) $-\frac{1}{2}\cos^2(2x) + C$
C) $\frac{1}{2}\sin(2x) + C$
D) $\frac{1}{2}\sin^2(2x) + C$
E) $-\frac{1}{2}\sin(2x) + C$

27. Find $\int_0^{\frac{\pi}{3}} \sin(3x)\, dx =$

A) -2 B) $-\frac{2}{3}$ C) 0 D) $\frac{2}{3}$ E) 2

28. Find $\int \sin(2x+3)dx =$

A) $-2\cos(2x+3) + C$
B) $-\cos(2x+3) + C$
C) $-\frac{1}{2}\cos(2x+3) + C$
D) $\frac{1}{2}\cos(2x+3) + C$
E) $\cos(2x+3) + C$

29. If $\frac{dy}{dx} = \sin x \cos^2 x$ and if $y = 0$ when $x = \frac{\pi}{2}$, what is the value of y when $x = 0$?

A) -1 B) $-\frac{1}{3}$ C) 0 D) $\frac{1}{3}$ E) 1

30. Find $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} =$

A) $\frac{\pi}{3}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{6}$ D) $\frac{1}{2}\ln 2$ E) $-\ln 2$

31. Which of the following is equal to $\int \frac{1}{\sqrt{25-x^2}}dx$?

A) $\arcsin\frac{x}{5} + C$
B) $\arcsin x + C$
C) $\frac{1}{5}\arcsin\frac{x}{5} + C$
D) $\sqrt{25-x^2} + C$
E) $2\sqrt{25-x^2} + C$

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32. An antiderivative for $\dfrac{1}{x^2-2x+2}$ is

A) $-\left(x^2-2x+2\right)^{-2}$
B) $\ln\left(x^2-2x+2\right)$
C) $\ln\left|\dfrac{x-2}{x+1}\right|$
D) $\sec^{-1}\left(x-1\right)$
E) $\tan^{-1}\left(x-1\right)$

33. An antiderivative for $f(x)=e^{x+e^x}$ is

A) $\dfrac{e^{x+e^x}}{1+e^x}$
B) $\left(1+e^x\right)e^{x+e^x}$
C) e^{1+e^x}
D) e^{x+e^x}
E) e^{e^x}

34. Find $\int xe^{2x}\,dx =$

A) $\dfrac{xe^{2x}}{2}-\dfrac{e^{2x}}{4}+C$
B) $\dfrac{xe^{2x}}{2}-\dfrac{e^{2x}}{2}+C$
C) $\dfrac{xe^{2x}}{2}+\dfrac{e^{2x}}{4}+C$
D) $\dfrac{xe^{2x}}{2}+\dfrac{e^{2x}}{2}+C$
E) $\dfrac{x^2e^{2x}}{4}+C$

35. Find $\int_0^1 x^3e^{x^4}\,dx =$

A) $\dfrac{1}{4}\left(e-1\right)$ B) $\dfrac{1}{4}e$ C) $e-1$ D) e E) $4\left(e-1\right)$

36. Find $\int \tan(2x)dx =$

A) $-2\ln\left|\cos(2x)\right|+C$
B) $-\dfrac{1}{2}\ln\left|\cos(2x)\right|+C$
C) $\dfrac{1}{2}\ln\left|\cos(2x)\right|+C$
D) $2\ln\left|\cos(2x)\right|+C$
E) $\dfrac{1}{2}\sec(2x)\tan(2x)+C$

37. Find $\int_2^3 \dfrac{x}{x^2+1}\,dx =$

A) $\dfrac{1}{2}\ln\dfrac{3}{2}$ B) $\dfrac{1}{2}\ln 2$ C) $\ln 2$ D) $2\ln 2$ E) $\dfrac{1}{2}\ln 5$

38. For $x > 0$, $\int \left(\frac{1}{x} \int_1^x \frac{du}{u} \right) dx =$

A) $\frac{1}{x^3} + C$

B) $\frac{8}{x^4} - \frac{2}{x^2} + C$

C) $\ln(\ln x) + C$

D) $\frac{\ln(x^2)}{2} + C$

E) $\frac{(\ln x)^2}{2} + C$

39. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$

A) 0.048 B) 0.144 C) 5.827 D) 23.308 E) 1,640.250

40. Find $\int_1^2 \frac{x+1}{x^2 + 2x} dx =$

A) $\ln 8 - \ln 3$

B) $\frac{\ln 8 - \ln 3}{2}$

C) $\ln 8$

D) $\frac{3 \ln 2}{2}$

E) $\frac{3 \ln 2 + 2}{2}$

41. If f is a continuous function and if $F'(x) = f(x)$ for all real numbers x, then $\int_1^3 f(2x)\, dx =$

A) $2F(3) - 2F(1)$

B) $\frac{1}{2}F(3) - \frac{1}{2}F(1)$

C) $2F(6) - 2F(2)$

D) $F(6) - F(2)$

E) $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

42. If the substitution $u = \frac{x}{2}$ is made, the integral $\int_2^4 \frac{1 - \left(\frac{x}{2}\right)^2}{x} dx =$

A) $\int_1^2 \frac{1-u^2}{u} du$

B) $\int_2^4 \frac{1-u^2}{u} du$

C) $\int_1^2 \frac{1-u^2}{2u} du$

D) $\int_1^2 \frac{1-u^2}{4u} du$

E) $\int_2^4 \frac{1-u^2}{2u} du$

Separation of Variables

43. If $\frac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

A) $-\frac{2}{3}$ B) $-\frac{1}{3}$ C) 0 D) $\frac{1}{3}$ E) $\frac{2}{3}$

44. If $\frac{dy}{dx} = -2y$ and if $y = 1$ when $x = 0$, what is the value of x for which $y = \frac{1}{2}$?

A) $-\frac{\ln 2}{2}$ B) $-\frac{1}{4}$ C) $\frac{\ln 2}{2}$ D) $\frac{\sqrt{2}}{2}$ E) $\ln 2$

45. At each point (x, y) on a certain curve, the slope of the curve is $3x^2 y$. If the curve contains the point (0, 8), then its equation is

A) $y = 8e^{x^3}$
B) $y = x^3 + 8$
C) $y = e^{x^3} + 7$
D) $y = \ln(x+1) + 8$
E) $y^2 = x^3 + 8$

46. If $\frac{dy}{dx} = y\sec^2 x$ and if $y = 5$ when $x = 0$, then $y =$

A) $e^{\tan x} + 4$
B) $e^{\tan x} + 5$
C) $5e^{\tan x}$
D) $\tan x + 5$
E) $\tan x + 5e^x$

47. If $\frac{dy}{dx} = x^2 y$, then y could be

A) $3\ln\left(\frac{x}{3}\right)$
B) $e^{\frac{x^3}{3}} + 7$
C) $2e^{\frac{x^3}{3}}$
D) $3e^{2x}$
E) $\frac{x^3}{3} + 1$

48. (1998 AB4) Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\frac{3x^2+1}{2y}$.

(a) Find the slope of the graph of f at the point where $x = 1$.
(b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.

(c) Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx}=\frac{3x^2+1}{2y}$ with the initial condition $f(1)=4$.

(d) Use your solution from part (c) to find $f(1.2)$.

49. (1985 BC4) Given the differential equation $\frac{dy}{dx}=\frac{-xy}{\ln y}, y > 0$.

(a) Find the general solution of the differential equation.

(b) Find the solution that satisfies the condition that $y=e^2$ when $x = 0$. Express your answer in the form $y=f(x)$.

(c) Explain why $x = 2$ is not in the domain of the solution found in part (b).

50. (2000 AB6) Consider the differential equation $\frac{dy}{dx}=\frac{3x^2}{e^{2y}}$.

(a) Find a solution $y=f(x)$ to the differential equation satisfying $f(0)=\frac{1}{2}$.

(b) Find the domain and range of the function *f* founding part (a).

51. (2002 BC 5) Consider the differential equation $\frac{dy}{dx}=\frac{3-x}{y}$.

(a) Let $y=f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y=-2$ is tangent to the graph of *f*. Find the *x*-coordinate of the point of tangency, and determine whether *f* has a local maximum, local minimum or neither at this point.

(b) Let $y=g(x)$ be the particular solution to the given differential equation for $-2<x<8$, with the initial condition $g(6)=-4$. Find $y=g(x)$.

52. (2001 AB6) The function *f* is differentiable for all real numbers. The point $\left(3,\frac{1}{4}\right)$ is on the graph of $y=f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx}=y^2(6-2x)$.

(a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3,\frac{1}{4}\right)$.

(b) Find $y=f(x)$ by solving the differential equation $\frac{dy}{dx}=y^2(6-2x)$ with the initial condition $f(3)=1/4$.

53. (2001 BC5) Let f be the function satisfying $f'(x) = -3xf(x)$, for all real numbers x, with $f(1) = 4$ and $\lim_{x\to\infty} f(x) = 0$.

(c) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = -3xy$ with the initial condition $f(1) = 4$. (5 points)

Integration by Parts

54. Which of the following is equal to ln4?

A) ln3+ln1

B) $\frac{\ln 8}{\ln 2}$

C) $\int_1^4 e^t\,dt$

D) $\int_1^4 \ln t\,dt$

E) $\int_1^4 \frac{1}{t}dt$

55. Find $\int_0^1 xe^{-x}dx =$

A) $1-2e$ B) -1 C) $1-2e^{-1}$ D) 1 E) $2e-1$

56. Find $\int_0^{\frac{\pi}{2}} x\cos x\,dx =$

A) $-\frac{\pi}{2}$ B) -1 C) $1-\frac{\pi}{2}$ D) 1 E) $\frac{\pi}{2}-1$

57. Find $\int x\sec^2 x\,dx =$

A) $x\tan x + C$

B) $\frac{x^2}{2}\tan x + C$

C) $\sec^2 x + 2\sec^2 x\tan x + C$

D) $x\tan x - \ln\left|\cos x\right| + C$

E) $x\tan x + \ln\left|\cos x\right| + C$

58. Find $\int x\,f(x)dx =$

A) $x\,f(x) - \int x\,f'(x)dx$

B) $\frac{x^2}{2}f(x) - \int \frac{x^2}{2}f'(x)dx$

C) $x\,f(x) - \frac{x^2}{2}f(x) + C$

D) $x\,f(x) - \int f'(x)dx$

E) $\frac{x^2}{2}\int f(x)dx$

59. If $\int f(x)\sin x\,dx = -f(x)\cos x + \int 3x^2\cos x\,dx$, then $f(x)$ could be

A) $3x^2$ B) x^3 C) $-x^3$ D) $\sin x$ E) $\cos x$

Integration by Partial Fractions

60. Find $\int \frac{dx}{(x-1)(x+2)} =$

A) $\frac{1}{3}\ln\left|\frac{x-1}{x+2}\right| + C$

B) $\frac{1}{3}\ln\left|\frac{x+2}{x-1}\right| + C$

C) $\frac{1}{3}\ln\left|(x-1)(x+2)\right| + C$

D) $\left(\ln|x-1|\right)\left(\ln|x+2|\right) + C$

E) $\ln\left|(x-1)(x+2)^2\right| + C$

61. Find $\int_2^3 \frac{3}{(x-1)(x+2)}\,dx =$

A) $-\frac{33}{20}$ B) $-\frac{9}{20}$ C) $\ln\left(\frac{5}{2}\right)$ D) $\ln\left(\frac{8}{5}\right)$ E) $\ln\left(\frac{2}{5}\right)$

62. Find $\int \frac{1}{x^2-6x+8}\,dx =$

A) $\frac{1}{2}\ln\left|\frac{x-4}{x-2}\right| + C$

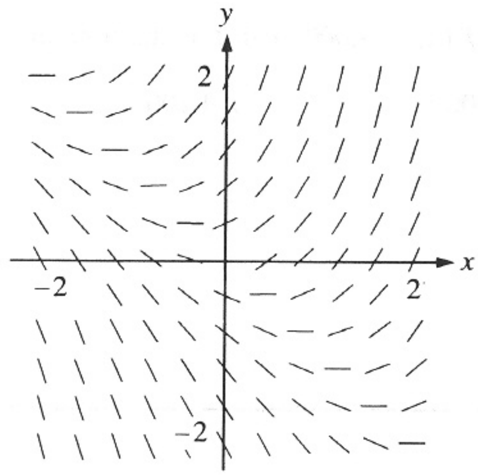
B) $\frac{1}{2}\ln\left|\frac{x-2}{x-4}\right| + C$

C) $\frac{1}{2}\ln\left|(x-2)(x-4)\right| + C$

D) $\frac{1}{2}\ln\left|(x-4)(x+2)\right| + C$

E) $\ln\left|(x-2)(x-4)\right| + C$

Slope Fields and Euler's Method



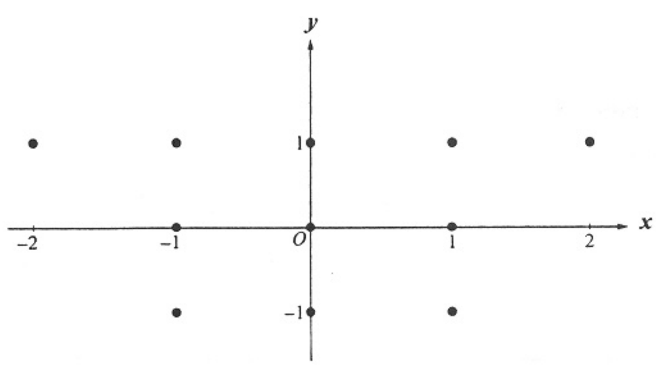
63. Shown above is a slope field for which of the following differential equations?

A) $\frac{dy}{dx} = 1 + x$ C) $\frac{dy}{dx} = x + y$ E) $\frac{dy}{dx} = \ln y$

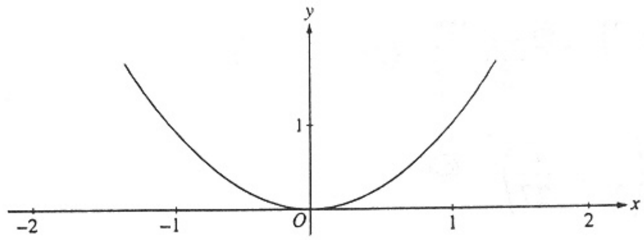
B) $\frac{dy}{dx} = x^2$ D) $\frac{dy}{dx} = \frac{x}{y}$

64. (2000 BC6) Consider the differential equation given by $\frac{dy}{dx} = x(y-1)^2$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eleven points indicated.



(b) Use the slope field for the given differential equation to explain why a solution could not have the graph shown below.

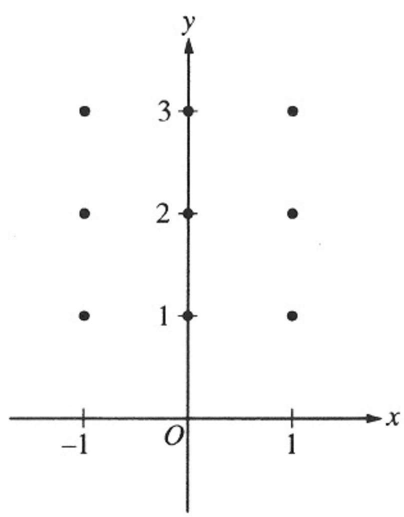


(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -1$.

(d) Find the range of the solution found in part (c).

65. (1998 BC4) Consider the differential equation given by $\dfrac{dy}{dx} = \dfrac{xy}{2}$.

(a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.

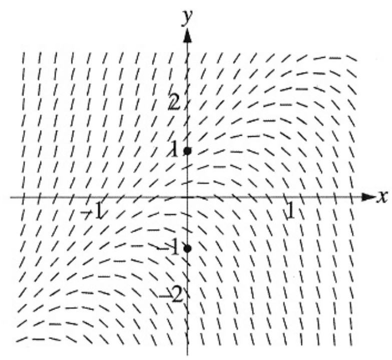


(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 3$. Use Euler's method starting at $x = 0$, with a step size of 0.1, to approximate $f(0.2)$. Show the work that leads to your answer.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$. Use your solution to find $f(0.2)$.

66. (2002 BC5) Consider the differential equation $\frac{dy}{dx} = 2y - 4x$.

(a) The slope field for the given differential equation is provided. Sketch the solution curve that passes through the point (0, 1) and sketch the solution curve that passes through the point (0, -1).



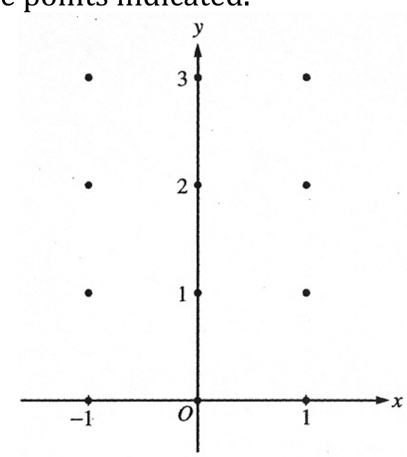
(b) Let f be the function that satisfies the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with a step size of 0.1 to approximate $f(0.2)$. Show the work that leads to your answer.

(c) Find the value of b for which $y = 2x + b$ is a solution to the given differential equation. Justify your answer.

(d) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. Does the graph of g have a local extreme at the point (0, 0)? If so, is the point a local maximum or a local minimum? Justify your answer.

67. (2004 AB6) Consider the differential equation $\frac{dy}{dx} = x^2(y - 1)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the *x*-*y* plane. Describe all points in the *x*-*y* plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

Exponential Growth and Decay

68. A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

A) 4.2 pounds
B) 4.6 pounds
C) 4.8 pounds
D) 5.6 pounds
E) 6.5 pounds

69. Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

A) 0.069 B) 0.200 C) 0.301 D) 3.322 E) 5.000

70. Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

A) $\frac{3\ln 3}{\ln 2}$ B) $\frac{2\ln 3}{\ln 2}$ C) $\frac{\ln 3}{\ln 2}$ D) $\ln\left(\frac{27}{2}\right)$ E) $\ln\left(\frac{9}{2}\right)$

71. During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

A) 343 B) 1,343 C) 1,367 D) 1,400 E) 2,057

72. The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$, where the initial population $P(0) = 3{,}000$ and t is the time in years. What is $\lim_{t \to \infty} P(t)$?

A) 2,500 B) 3,000 C) 4,200 D) 5,000 E) 10,000

73. (1989 AB6) Oil is being pumped continuously from a certain oil well at a rate proportional to the amount of oil left in the well; that is, $\frac{dy}{dt} = ky$, where *y* is the amount of oil left in the well at any time *t*. Initially there were 1,000,000 gallons of oil in the well and 6 years later there were 500,000 gallons remaining. It will no longer be profitable to pump oil when there are fewer than 50,000 gallons remaining.

(a) Write an equation for *y*, the amount of oil remaining in the well at any time *t*.
(b) At what rate is the amount of oil in the well decreasing when there are 600,000 gallons of oil remaining?
(c) In order not to lose money, at what time *t* should oil no longer be pumped from the well?

74. (1996 BC3) The rate of consumption of cola in the United States is given by $S(t) = Ce^{kt}$, where *S* is measured in billions of gallons per year and *t* is measured in years from the beginning of 1980.

(a) The consumption rate doubles every 5 years and the consumption rate at the beginning of 1980 was 6 billion gallons per year. Find *C* and *k*.
(b) Find the average rate of consumption of cola over the 10-year time period beginning January 1, 1983. Indicate units of measure.
(c) Use the trapezoidal rule with four equal subdivisions to estimate $\int_5^7 S(t)dt$.
(d) Using correct units, explain the meaning of $\int_5^7 S(t)dt$ in terms of cola consumption.

75. (1974 AB7) The rate of change in the number of bacteria in a culture is proportional to the number present. In a certain laboratory experiment, a culture had 10,000 bacteria initially, 20,000 bacteria at time t_1 minutes, and 100,000 bacteria at $t_1 + 10$ minutes.

(a) In terms of *t* only, find the number of bacteria in the culture at any time *t* minutes, $t \geq 0$.
(b) How many bacteria were there after 20 minutes?
(c) How many minutes had elapsed when the 20,000 bacteria were observed?

76. (1987 BC1) At any time $t \geq 0$, in days, the rate of growth of a bacteria population is given by $y' = ky$, where *k* is a constant and *y* is the number of bacteria present. The initial population is 1,000 and the population triples during the first 5 days.

(a) Write an expression for *y* at any time $t \geq 0$.
(b) By what factor will the population have increased in the first 10 days?
(c) At what time *t*, in days, will the population have increased by a factor of 6?

77. (2004 BC5) A population is modeled by a function P that satisfies the logistic differential equation $\frac{dP}{dt}=\frac{P}{5}\left(1-\frac{P}{12}\right)$.

(a) If $P(0)=3$, what is $\lim_{t\to\infty}P(t)$? If $P(0)=20$, what is $\lim_{t\to\infty}P(t)$?

(b) If $P(0)=3$, for what value of P is the population growing the fastest?

(d) A different population is modeled by a function Y that satisfies the separable differential equation $\frac{dY}{dt}=\frac{Y}{5}\left(1-\frac{t}{12}\right)$. Find $Y(t)$ if $Y(0)=3$.

(d) For the function Y found in part (c), what is $\lim_{t\to\infty}Y(t)$?

Antiderivatives

1.	E	1993	AB	#17	65%
2.	A	1993	AB	#38	82%
3.	A	1988	AB	#5	89%

Evaluating Definite Integrals

4.	A	1998	AB	#11	42%
5.	D	1985	AB	#1	89%
6.	D	1985	AB	#24	70%
7.	C	1998	AB	#3	71%
8.	E	1998	AB	#7	43%
9.	D	1988	AB	#17	72%
10.	E	1985	BC	#36	26%
11.	A	1985	AB	#22	66%
12.	C	1988	AB	#10	80%
13.	A	1998	AB	#20	69%
14.	A	1993	BC	#33	73%
15.	D	1985	AB	#27	25%
16.	C	1988	AB	#28	34%
17.	A	1988	AB	#13	70%
18.	B	1993	BC	#37	71%
19.	B	1993	AB	#28	13%
20.	E	1993	BC	#3	82%
21.		1988	AB	#6	

U-Substitutions

22.	D	1988	AB	#7	79%
23.	D	1988	AB	#14	67%
24.	A	1993	AB	#14	69%
25.	D	1988	BC	#2	89%
26.	C	1985	AB	#4	81%
27.	D	1985	AB	#32	56%
28.	C	1985	BC	#18	89%
29.	B	1998	BC	#8	55%
30.	A	1993	AB	#32	29%
31.	A	1985	BC	#7	57%
32.	E	1993	AB	#22	32%
33.	E	1985	BC	#28	35%
34.	A	1988	BC	#16	80%
35.	A	1993	BC	#7	81%
36.	B	1985	AB	#30	55%
37.	B	1988	AB	#19	58%
38.	E	1988	AB	#38	47%
39.	C	1998	AB	#88	55%
40.	B	1985	BC	#3	90%
41.	E	1998	AB	#82	24%
42.	A	1985	BC	#40	47%

Separation of Variables

43.	B	1993	AB	#33	14%
44.	C	1985	BC	#33	48%
45.	A	1985	BC	#44	52%
46.	C	1988	BC	#39	43%
47.	C	1993	BC	#13	34%
48.		1998	AB	#4	
49.		1985	BC	#4	
50.		2000	AB	#6	
51.		2002	BC	#5	FormB
52.		2001	AB	#6	
53.		2001	BC	#5	

Integration by Parts

54.	E	1985	AB	#7	62%
55.	C	1985	AB	#17	42%
56.	E	1988	AB	#26	59%
57.	E	1993	BC	#29	61%
58.	B	1993	AB	#43	46%
59.	B	1985	BC	#21	61%

Integration by Partial Fractions

60.	A	1985	BC	#12	60%
61.	D	1988	BC	#17	70%
62.	A	1998	BC	#4	61%

Slope Fields and Euler's Method

63.	C	1998	BC	#24	38%
64.		2000	BC	#6	
65.		1998	BC	#4	
66.		2002	BC	#5	
67.		2004	AB	#6	

Exponential Growth and Decay

68.	B	1993	AB	#42	30%
69.	A	1998	AB	#84	42%
70.	A	1988	BC	#43	44%
71.	C	1993	BC	#38	63%
72.	E	1998	BC	#26	20%
73.		1989	AB	#6	FRQ
74.		1996	BC	#3	FRQ
75.		1974	AB	#7	FRQ
76.		1987	BC	#1	FRQ
77.		2004	BC	#5	FRQ