AP Calculus BC
Section 6.5 – Logistic Growth FDWK

For problems 1 – 4, find *k* and the carrying capacity for the population represented by the differential equation.

1. $\frac{dP}{dt} = 0.04P - 0.0004P^2$

2. $\frac{50}{P}\frac{dP}{dt} = 2 - \frac{P}{250}$

3. $\frac{dB}{dt} = 0.06B\left(1 - \frac{B}{100}\right)$

4. $50\frac{dY}{dt} = 3Y - 0.03Y^2$

5. (#17) A 2000-gallon tank can support no more than 150 guppies. Six guppies are introduced into the tank. Assume the rate of growth of the population is $\frac{dP}{dt} = 0.0015P(150 - P)$.

   a. What is *k* and the carrying capacity?

   b. When is the population growing the fastest?

   c. What is $\lim_{t\to\infty} P(t)$?

   d. On what interval(s) is the population increasing at an increasing rate?

   e. What is the largest rate of growth of the guppy population?

   f. Find an expression for the population of guppies at any given time, *t*.

   g. How long will it take for the guppy population to reach 100?

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6. (#18) A certain wild animal preserve can support no more than 250 lowland gorillas. Twenty-eight gorillas were know to be in the preserve in 1970. Assume the rate of growth of the population is $\frac{dP}{dt} = 0.0004P(250 - P)$.

   a. What is k and the carrying capacity?

   b. When is the population growing the fastest?

   c. What is $\lim_{t \to \infty} P(t)$?

   d. On what interval(s) is the population increasing at a decreasing rate?

   e. What is the largest rate of growth of the gorilla population?

   f. Find an expression for the population of gorillas at any given time, t.

   g. How long will it take for the gorilla population to reach 225?

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$\frac{dP}{dt} = \frac{k}{M}P(M-P) = kP - \frac{k}{M}P^2$ M = ZERO OF DIFF. EQ.

For problems 1 – 4, find k and the carrying capacity for the population represented by the differential equation.

1. $\frac{dP}{dt} = 0.04P - 0.0004P^2$ (k = .04) $= .0004P(100-P)$ $M = 100$, $k = .04$

2. $\frac{50}{P}\frac{dP}{dt} = \left(2 - \frac{P}{250}\right)\frac{P}{50}$
$= \frac{1}{50}P\cdot\frac{1}{250}(500-P)$
$= \frac{.04}{500}P(500-P)$
$M = 500$, $k = .04$

3. $\frac{dB}{dt} = 0.06B\left(1 - \frac{B}{100}\right)$
$= \frac{.06}{100}B(100-B)$
$M = 100$, $k = .06$

4. $50\frac{dY}{dt} = (3Y - 0.03Y^2)\frac{1}{50}$
$= .03Y(100-Y)\cdot\frac{1}{50}$
$\frac{.03}{50} = \frac{.06}{100}$
$M = 100$, $k = .06$

5. (#17) A 2000-gallon tank can support no more than 150 guppies. Six guppies are introduced into the tank. Assume the rate of growth of the population is $\frac{dP}{dt} = 0.0015P(150-P)$. t IN WEEKS
$\frac{k}{M} = .0015$
$k = .0015(150) = .225$

   a. What is k and the carrying capacity?
   $M = 150$, $k = .225$

   b. When is the population growing the fastest? $P = 75$

   c. What is $\lim_{t\to\infty} P(t)$? 150

   d. On what interval(s) is the population increasing at an increasing rate? $0 < P < 75$

   e. What is the largest rate of growth of the guppy population? $\left.\frac{dP}{dt}\right|_{P=75} = .0015(75)(75) = 8.4375$ GUPPIES/WK

   f. Find an expression for the population of guppies at any given time, t.
   $P = \frac{150}{1+Ae^{-.225t}}$
   $6 = \frac{150}{1+A}$
   $1+A = \frac{150}{6} = 25$
   $A = 24$
   $P = \frac{150}{1+24e^{-.225t}}$

   g. How long will it take for the guppy population to reach 100?
   $100 = \frac{150}{1+24e^{-kt}}$
   $1+24e^{-kt} = \frac{150}{100}$
   $24e^{-kt} = \frac{150}{100} - 1$
   $-kt = \ln\left(\frac{.5}{24}\right)$
   $t = \frac{\ln\left(\frac{.5}{24}\right)}{-k} \approx 17.205$ WEEKS

$\frac{dP}{dt} = \frac{k}{m}P(m-P) =$

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LET $t=0$ BE 1970.

6. (#18) A certain wild animal preserve can support no more than 250 lowland gorillas. Twenty-eight gorillas were know to be in the preserve in 1970. Assume the rate of growth of the population is $\frac{dP}{dt} = 0.0004P(250-P)$. * t IN YEARS

a. What is k and the carrying capacity? $m = 250$ $\frac{k}{250} = .0004 \Rightarrow k = .1$

b. When is the population growing the fastest? $P = 125$

c. What is $\lim_{t\to\infty} P(t)$? 250

d. On what interval(s) is the population increasing at a decreasing rate?

$125 < P < 250$

e. What is the largest rate of growth of the gorilla population?

$\left.\frac{dP}{dt}\right|_{P=125} = .0004(125)(250-125) = 6.25 \frac{\text{GORILLAS}}{\text{YR}}$

f. Find an expression for the population of gorillas at any given time, t.

$P = \frac{250}{1+Ae^{-.1t}}$ $A \approx 7.9285$ $P = \frac{250}{1+7.9285e^{-.1t}}$

$28 = \frac{250}{1+A}$

$1+A = \frac{250}{28} - 1$

g. How long will it take for the gorilla population to reach 225?

$225 = \frac{250}{1+7.9285e^{-.1t}}$ IN ABOUT THE YEAR 2012

$t = \frac{\ln\left(\frac{\frac{250}{225}-1}{A}\right)}{-.1} \approx 42.6769$