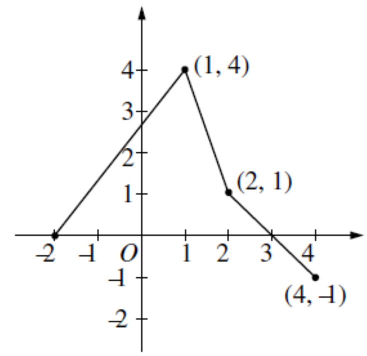
AP Calculus BC
Chapter 6 Test 1 Review – FTC FRQ

1. (1999 BC5) The graph of the function *f*, consisting of three line segments, is given. Let $g(x)=\int_1^x f(t)dt$.

   a. Compute $g(4)$ and $g(-2)$.



   b. Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

   c. Find the average rate of change of g over the interval [–2, 4].

   d. Find the average value of g' over the interval [1, 2].

   e. Find the minimum value of g on the closed interval [–2, 4]. Justify your answer.

   f. The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x–coordinates of points of inflection of the graph of g? Justify your answer.

   g. Find the equation of the tangent line to g at x = 2.

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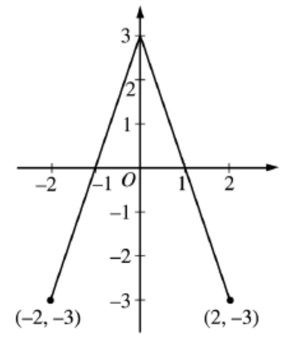
2. (2002 BC4) The graph of the function f shown consists of two line segments. Let g be the function given by $g(x)=\int_0^x f(t)\,dt$.



Graph of f

a. Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

b. For what values of x in the open interval (–2, 2) is g increasing? Explain your reasoning.

c. For what values of x in the open interval (–2, 2) is the graph of g concave down? Explain your reasoning.

d. Sketch a graph of g on the closed interval [–2, 2].

e. Find the average value of g' on [0, 1].

f. Find the instantaneous rate of change of g' at $x = 1$.

g. Find the equation of the tangent line to g' at $x = -1$.

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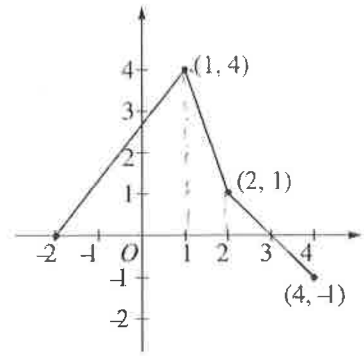
1. (1999 BC5) The graph of the function f, consisting of three line segments, is given. Let $g(x)=\int_1^x f(t)dt$.



a. Compute $g(4)$ and $g(-2)$.

$g(4)=\int_1^4 f(t)dt = \boxed{5/2}$

$g(-2)=\int_1^{-2} f(t)dt = -\int_{-2}^1 f(t)dt = \boxed{-6}$

b. Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

$g'(x) = f(x)$

$g'(1) = f(1) = \boxed{4}$

c. Find the average rate of change of g over the interval [−2, 4].

$\frac{g(4)-g(-2)}{4-(-2)} = \frac{5/2+6}{6} = \frac{5+12}{12} = \boxed{17/12}$

d. Find the average value of g' over the interval [1, 2].

$\frac{1}{2-1}\int_1^2 g'(x)dx = g(x)\Big]_1^2 = (g(2)-g(1)) = \boxed{5/2}$

$g(2) = 5/2$

e. Find the minimum value of g on the closed interval [−2, 4]. Justify your answer.

$g' = f = 0$ AT $x=3$

$g(-2) = -6$

$g(3) = 3$

$g(4) = 5/2$

$\boxed{\text{MIN VALUE IS } -6.}$

f. The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x–coordinates of points of inflection of the graph of g? Justify your answer.

$g'' = f'$ CHANGES SIGN AT $x=1$ BUT NOT AT $x=2 \Rightarrow g$ HAS POINT OF INFECTION AT $x=1$ ONLY.

g. Find the equation of the tangent line to g at x = 2.

$g(2) = 5/2$

$g'(2) = f(2) = 1$

$\boxed{y - 5/2 = 1(x-2)}$

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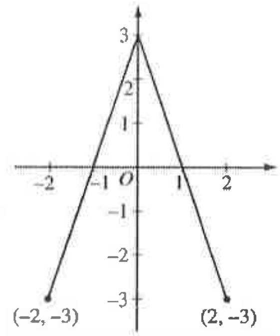


Graph of f

a. Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

$g(-1) = \int_0^{-1} f(t)\,dt = -\int_{-1}^{0} f(t)\,dt = \boxed{-3/2}$ $\qquad g''(-1) = f'(-1) = \boxed{3}$

$g'(-1) = f(-1) = \boxed{0}$

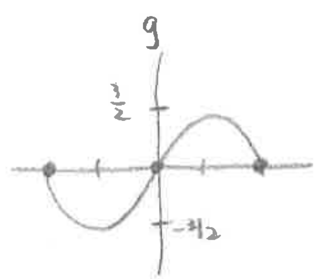
b. For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.

g INCR IF $g' = f > 0$, SO g INCR. ON $[-1, 1]$.

c. For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.

$g'' = f' < 0$ on $(0, 2) \Rightarrow g$ CONCAVE DOWN on $(0, 2)$.

d. Sketch a graph of g on the closed interval $[-2, 2]$.



e. Find the average value of g' on $[0, 1]$.

$\frac{1}{1-0}\int_0^1 g'(x)\,dx = g(x)\Big]_0^1 = g(1) - g(0) = \frac{3}{2} - 0 = \boxed{\frac{3}{2}}$

f. Find the instantaneous rate of change of g' at $x = 1$.

$g''(1) = f'(1) = \boxed{-3}$

g. Find the equation of the tangent line to g' at $x = -1$.

$g'(-1) = f(-1) = 0$ $\qquad \boxed{y - 0 = 3(x+1)}$

$g''(-1) = f'(-1) = 3$