

**AP Calculus BC**  
**Section 8.4 – Partial Fractions (Anton)**

Evaluate the following integrals.

1.  $\int \frac{dx}{x^2 + 4x + 3}$

2.  $\int \frac{dx}{x^2 + 8x + 7}$

3.  $\int \frac{x}{x^2 - 5x + 6} dx$

4.  $\int \frac{5x - 4}{x^2 - 4x} dx$

5.  $\int \frac{11x + 17}{2x^2 + 7x - 4} dx$

6.  $\int \frac{5x - 5}{3x^2 - 8x - 3} dx$

7.  $\int \frac{dx}{x(x^2 - 1)}$

8.  $\int \frac{2x^2 - 9x - 9}{x^3 - 9x} dx$

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9.  $\int \frac{2x^2 + 4x - 8}{x^3 - 4x} dx$

10.  $\int \frac{x^2 + 2}{x + 2} dx$

11.  $\int \frac{x^2 - 4}{x - 1} dx$

12.  $\int \frac{3x^2 - 10}{x^2 - 4x + 4} dx$

13.  $\int \frac{x^2}{x^2 - 3x + 2} dx$

14.  $\int \frac{x^3}{x^2 - 3x + 2} dx$

15.  $\int \frac{x^3}{x^2 - x - 6} dx$

16.  $\int \frac{x^5 + 2x^2 + 1}{x - x^3} dx$

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Evaluate the following integrals.

$$1. \int \frac{dx}{x^2+8x+4} = \int \frac{-1/2}{x+3} + \frac{1/2}{x+1} dx$$

$(x+3)(x+1)$

$$= \frac{1}{2} \ln|x+1| - \frac{1}{2} \ln|x+3| + C$$

$$= \frac{1}{2} \ln \left| \frac{x+1}{x+3} \right| + C$$

$$2. \int \frac{dx}{x^2+8x+7} = \int \frac{-1/6}{x+7} + \frac{1/6}{x+1} dx$$

$(x+7)(x+1)$

$$= \frac{1}{6} \ln|x+1| - \frac{1}{6} \ln|x+7| + C$$

$$= \frac{1}{6} \ln \left| \frac{x+1}{x+7} \right| + C$$

$$3. \int \frac{x}{x^2-5x+6} dx = \int \frac{-2}{x-2} + \frac{3}{x-3} dx$$

$(x-2)(x-3)$

$$= 3 \ln|x-3| - 2 \ln|x-2| + C$$

$$4. \int \frac{5x-4}{x^2-4x} dx = \int \frac{1}{x} + \frac{4}{x-4} dx$$

$x(x-4)$

$$= \ln|x| + 4 \ln|x-4| + C$$

$$5. \int \frac{11x+17}{2x^2+7x-4} dx = \int \frac{5}{2x-1} + \frac{3}{x+4} dx$$

$(2x-1)(x+4)$

$$= \frac{5}{2} \ln|2x-1| + 3 \ln|x+4| + C$$

$$6. \int \frac{5x-5}{3x^2-8x-3} dx = \int \frac{2}{3x+1} + \frac{1}{x-3} dx$$

$(3x+1)(x-3)$

$$= \frac{2}{3} \ln|3x+1| + \ln|x-3| + C$$

$$7. \int \frac{dx}{x(x^2-1)} = \int \frac{-1}{x} + \frac{1/2}{x-1} + \frac{1/2}{x+1} dx$$

$x(x-1)(x+1)$

$$= \frac{1}{2} \ln|x-1| + \frac{1}{2} \ln|x+1| - \ln|x| + C$$

$$= \frac{1}{2} \ln|x^2-1| - \ln|x| + C$$

$$8. \int \frac{2x^2-9x-9}{x^3-9x} dx = \int \frac{1}{x} + \frac{-1}{x-3} + \frac{2}{x+3} dx$$

$x(x-3)(x+3)$

$$= \ln|x| - \ln|x-3| + 2 \ln|x+3| + C$$

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$$9. \int \frac{2x^2 + 4x - 8}{x^3 - 4x} dx = \int \frac{2}{x} + \frac{1}{x-2} + \frac{-1}{x+2}$$

$$x(x-2)(x+2)$$

$$2\ln|x| + \ln|x-2| - \ln|x+2| + C$$

$$10. \int \frac{x^2 + 2}{x+2} dx = \int x-2 + \frac{6}{x+2} dx$$

$$x+2 \overline{) \begin{array}{r} x-2 \\ x^2+0x+2 \\ \hline x^2+2x \\ \hline -2x+2 \\ -2x-4 \\ \hline 6 \end{array}}$$

$$= \frac{1}{2}x^2 - 2x + 6\ln|x+2| + C$$

$$11. \int \frac{x^2 - 4}{x-1} dx = \int x+1 - \frac{3}{x-1} dx$$

$$x-1 \overline{) \begin{array}{r} x+1 \\ x^2+0x-4 \\ \hline x^2-x \\ \hline x-4 \\ x-1 \\ \hline -3 \end{array}}$$

$$\frac{1}{2}x^2 + x - 3\ln|x-1| + C$$

$$12. \int \frac{3x^2 - 10}{x^2 - 4x + 4} dx = \int 3 + \frac{12x-22}{(x-2)^2} dx$$

$$x^2-4x+4 \overline{) \begin{array}{r} 3 \\ 3x^2+0x-10 \\ \hline 3x^2-12x+12 \\ \hline 12x-22 \end{array}}$$

$$= \int 3 + \frac{12}{x-2} + \frac{-2}{(x-2)^2}$$

$$3x + 12\ln|x-2| - \frac{2}{x-2} + C$$

$$A(x-2) + B = 12x-22$$

$$x=3: A+2=14 \Rightarrow A=12$$

$$13. \int \frac{x^2}{x^2 - 3x + 2} dx = \int 1 + \frac{3x-2}{(x-2)(x-1)} dx$$

$$x^2-3x+2 \overline{) \begin{array}{r} 1 \\ x^2+0x+0 \\ \hline x^2-3x+2 \\ \hline 3x-2 \end{array}}$$

$$= \int 1 + \frac{4}{x-2} + \frac{-1}{x-1} dx$$

$$x + 4\ln|x-2| - \ln|x-1| + C$$

$$14. \int \frac{x^3}{x^2 - 3x + 2} dx = \int x+3 + \frac{7x-6}{(x-2)(x-1)} dx$$

$$x^2-3x+2 \overline{) \begin{array}{r} x+3 \\ x^3+0x^2+0x+0 \\ \hline x^3-3x^2+2x \\ \hline 3x^2-2x+0 \\ 3x^2-9x+6 \\ \hline 7x-6 \end{array}}$$

$$= \int x+3 + \frac{8}{x-2} + \frac{-1}{x-1} dx$$

$$= \frac{1}{2}x^2 + 3x + 8\ln|x-2| - \ln|x-1| + C$$

$$15. \int \frac{x^3}{x^2 - x - 6} dx = \int x+1 + \frac{7x+6}{(x-3)(x+2)}$$

$$x^2-x-6 \overline{) \begin{array}{r} x+1 \\ x^3+0x^2+0x+0 \\ \hline x^3-x^2-6x \\ \hline x^2+6x+0 \\ x^2-x-6 \\ \hline 7x+6 \end{array}}$$

$$= \int x+1 + \frac{27/5}{x-3} + \frac{8/5}{x+2} dx$$

$$\frac{1}{2}x^2 + x + \frac{27}{5}\ln|x-3| + \frac{8}{5}\ln|x+2| + C$$

$$16. \int \frac{x^5 + 2x^2 + 1}{x - x^3} dx = \int -x^2 - 1 + \frac{2x^2 + x + 1}{x(1-x)(1+x)}$$

$$-x^3+x \overline{) \begin{array}{r} -x^2-1 \\ x^5+0x^4+0x^3+2x^2+0x+1 \\ \hline x^5-x^3 \\ \hline x^3+2x^2+0x+1 \\ x^3-x \\ \hline 2x^2+x+1 \end{array}}$$

$$= \int -x^2 - 1 + \frac{1}{x} + \frac{2}{1-x} + \frac{-1}{1+x} dx$$

$$-\frac{1}{3}x^3 - x + \ln|x| - 2\ln|1-x| - \ln|1+x| + C$$