AP Calculus BC
Section 6.3 – Integration by Parts (FDWK)

Evaluate the following integrals.

1. $\int x\sec^2 x\,dx$

2. $\int \sin^{-1}\theta\,d\theta$

3. $\int t^2 \sin t\,dt$

4. $\int z\csc^2 z\,dz$

5. $\int x^3 \ln x\,dx$

6. $\int x^4 e^{-x}\,dx$

7. $\int \left(x^2 - 5x\right)e^x\,dx$

8. $\int x^3 e^{-2x}\,dx$

AP Calculus BC
Section 6.3 – Integration by Parts (FDWK)

9. $\int e^{y} \sin y dy$

10. $\int e^{-x} \cos x dx$

11. $\int_0^{\pi/2} x^2 \sin 2x dx$

12. $\int_0^{\pi/2} x^3 \cos 2x dx$

Solve the following for *y*.

13. $\frac{dy}{dx} = x^2 e^{4x}$

14. $\frac{dy}{dx} = x^2 \ln x$

15. $\frac{dy}{d\theta} = \theta \sec^{-1} \theta$

16. $\frac{dy}{d\theta} = \theta \sec\theta \tan\theta$

AP Calculus BC
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Evaluate the following integrals.

1. $\int x\sec^2 x\,dx$ $\quad u=x \quad dv=\sec^2 x\,dx$ $\quad du=dx \quad v=\tan x$

$x\tan x-\int \tan x\,dx$ $\quad u=\cos x \quad du=-\sin x\,dx$

$x\tan x+\ln|\cos x|+C$

2. $\int \sin^{-1}\theta\,d\theta$ $\quad u=\sin^{-1}\theta \quad dv=d\theta$ $\quad du=\frac{1}{\sqrt{1-\theta^2}}d\theta \quad v=\theta$

$\theta\sin^{-1}\theta-\int\frac{\theta}{\sqrt{1-\theta^2}}d\theta$ $\quad u=1-\theta^2 \quad du=-2\theta\,d\theta$

$\theta\sin^{-1}\theta+\frac{1}{2}\int u^{-1/2}du$

$\theta\sin^{-1}\theta+\sqrt{1-\theta^2}+C$

3. $\int t^2\sin t\,dt$ $= -t^2\cos t-2t\sin t+2\cos t+C$

	u	dv
+	t^2	$\sin t$
−	$2t$	$-\cos t$
+	2	$-\sin t$
	0	$\cos t$

4. $\int z\csc^2 z\,dz$ $\quad u=z \quad dz=\csc^2 z$ $\quad du=dz \quad z=-\cot z$

$-z\cot z+\int\cot z\,dz$ $\quad u=\sin x \quad du=\cos x$

$-z\cot z+\ln|\sin x|+C$

5. $\int x^3\ln x\,dx$ $\quad u=\ln x \quad dv=x^3dx$ $\quad du=\frac{1}{x} \quad v=\frac{1}{4}x^4$

$\frac{1}{4}x^4\ln x-\frac{1}{4}\int x^3dx$

$\frac{1}{4}x^4\ln x-\frac{1}{16}x^4+C$

6. $\int x^4e^{-x}dx$ $= -x^4e^{-x}-4x^3e^{-x}-12x^2e^{-x}-24xe^{-x}-24e^{-x}+C$

	u	dv
+	x^4	e^{-x}
−	$4x^3$	$-e^{-x}$
+	$12x^2$	e^{-x}
−	$24x$	$-e^{-x}$
+	24	e^{-x}

7. $\int(x^2-5x)e^x dx$ $= (x^2-5x)e^x-(2x-5)e^x+2e^x+C$

	u	dv
+	x^2-5x	e^x
−	$2x-5$	e^x
+	2	e^x
	0	e^x

8. $\int x^3e^{-2x}dx$ $= -e^{-2x}\left(\frac{1}{2}x^3+\frac{3}{4}x^2+\frac{3}{4}x+\frac{3}{8}\right)+C$

	u	dv
+	x^3	e^{-2x}
−	$3x^2$	$-\frac{1}{2}e^{-2x}$
+	$6x$	$\frac{1}{4}e^{-2x}$
−	6	$-\frac{1}{8}e^{-2x}$
+	0	$\frac{1}{16}e^{-2x}$

AP Calculus BC
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9. $\int e^y \sin y\,dy$ $= -e^y\cos y + e^y\sin y - \int e^y\sin y\,dy$

	u	dv
+	e^y	$\sin y$
−	e^y	$-\cos y$
+	$\int e^y$	$-\sin y$

$2\int e^y\sin y\,dy = -e^y(\cos y - \sin y) + C$

$\int e^y\sin y\,dy = \boxed{\frac{-e^y(\cos y - \sin y)}{2} + C}$

10. $\int e^{-x}\cos x\,dx$ $= e^{-x}\sin x - e^{-x}\cos x - \int e^{-x}\cos x\,dx$

	u	dv
+	e^{-x}	$\cos x$
−	$-e^{-x}$	$\sin x$
+	$\int e^{-x}$	$-\cos x$

$2\int e^{-x}\cos x\,dx = \boxed{\frac{e^{-x}(\sin x - \cos x)}{2} + C}$

11. $\int_0^{\pi/2} x^2\sin 2x\,dx = -\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\Big]_0^{\pi/2}$

	u	dv
+	x^2	$\sin 2x$
−	$2x$	$-\frac{1}{2}\cos 2x$
+	2	$-\frac{1}{4}\sin 2x$
−	0	$\frac{1}{8}\cos 2x$

$-\frac{1}{2}\frac{\pi^2}{4}(-1) + 0 - \frac{1}{4} - (0+0+\frac{1}{4})$

$\boxed{\frac{\pi^2}{8} - \frac{1}{2}}$

12. $\int_0^{\pi/2} x^3\cos 2x\,dx$ $= \frac{1}{2}x^3\sin 2x + \frac{3}{4}x^2\cos 2x - \frac{3}{4}\sin 2x - \frac{3}{8}\cos 2x\Big]_0^{\pi/2}$

	u	dv
+	x^3	$\cos 2x$
−	$3x^2$	$\frac{1}{2}\sin 2x$
+	$6x$	$-\frac{1}{4}\cos 2x$
−	6	$-\frac{1}{8}\sin 2x$
+	0	$\frac{1}{16}\cos 2x$

$\left(0 + \frac{3}{4}\cdot\frac{\pi^2}{4}\cdot(-1) - 0 + \frac{3}{8}\right) - \left(-\frac{3}{8}\right)$

$\boxed{\frac{-3\pi^2}{16} + \frac{3}{4}}$

Solve the following for *y*.

13. $\frac{dy}{dx} = \int x^2e^{4x}$ $= \boxed{\frac{1}{4}x^2e^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{32}e^{4x} + C}$

	u	dv
+	x^2	e^{4x}
−	$2x$	$\frac{1}{4}e^{4x}$
+	2	$\frac{1}{16}e^{4x}$
−	0	$\frac{1}{64}e^{4x}$

14. $\frac{dy}{dx} = \int x^2\ln x$ $u = \ln x$ $dv = x^2\,dx$ $du = \frac{1}{x}dx$ $v = \frac{1}{3}x^3$

$\frac{1}{3}x^3\ln x - \frac{1}{3}\int x^2\,dx$

$\boxed{\frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C}$

15. $\frac{dy}{d\theta} = \int \theta\sec^{-1}\theta$ $u = \sec^{-1}\theta$ $dv = \theta\,d\theta$ $du = \frac{d\theta}{\theta\sqrt{\theta^2-1}}$ $v = \frac{1}{2}\theta^2$

$\frac{1}{2}\theta\sec^{-1}\theta - \frac{1}{2}\int \frac{\theta}{\sqrt{\theta^2-1}}\,d\theta$ $u = \theta^2 - 1$ $du = 2\theta\,d\theta$

$\frac{1}{2}\theta\sec^{-1}\theta - \frac{1}{4}\int u^{-1/2}\,d\theta$

$\boxed{\frac{1}{2}\theta\sec^{-1}\theta - \frac{1}{2}\sqrt{\theta^2-1} + C}$

16. $\frac{dy}{d\theta} = \theta\sec\theta\tan\theta$

	u	dv
+	θ	$\sec\theta\tan\theta$
−	1	$\sec\theta$
+	0	$\ln\lvert\sec\theta + \tan\theta\rvert$

$\boxed{\theta\sec\theta - \ln\lvert\sec\theta + \tan\theta\rvert + C}$