AP Calculus BC
Section 6.2 – Inverse Trig Integrals

Evaluate the following integrals.

1. $\int \frac{dx}{\sqrt{1-4x^2}}$

2. $\int \frac{dx}{1+16x^2}$

3. $\int \frac{dx}{x\sqrt{9x^2-1}}$

4. $\int \frac{e^x}{1+e^{2x}}dx$

5. $\int_{\ln 2}^{\ln\left(2/\sqrt{3}\right)} \frac{e^{-x}dx}{\sqrt{1-e^{-2x}}}$

6. $\int_1^3 \frac{dx}{\sqrt{x}\left(x+1\right)}$

7. $\int \frac{t}{t^4+1}dt$

8. $\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}dx$

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9. $\int \frac{\sin\theta}{\cos^2\theta+1}d\theta$

10. $\int \frac{dx}{x\sqrt{1-(\ln x)^2}}$

11. $\int \frac{dx}{\sqrt{9-x^2}}$

12. $\int \frac{dx}{5+x^2}$

13. $\int \frac{dx}{x\sqrt{x^2-\pi}}$

14. $\int \frac{e^x}{4+e^{2x}}dx$

Use completing the square to help evaluate the following.

15. $\int \frac{dx}{x^2-2x+2}$

16. $\int \frac{dx}{x^2+4x+13}$

17. $\int \frac{dx}{\sqrt{-x^2-4x}}$

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Evaluate the following integrals.

1. $\int \frac{dx}{\sqrt{1-4x^2}}$

$u=2x$
$du=2\,dx$

$\frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du$

$\boxed{\frac{1}{2}\sin^{-1}(2x)+C}$

2. $\int \frac{dx}{1+16x^2}$

$u=4x$
$du=4\,dx$

$\frac{1}{4}\int \frac{1}{1+u^2}\,du$

$\boxed{\frac{1}{4}\tan^{-1}(4x)+C}$

3. $\int \frac{dx}{x\sqrt{9x^2-1}}$

$u=3x$
$du=3\,dx$

$\frac{1}{3}\int \frac{du}{u/3\sqrt{u^2-1}}$

$\int \frac{du}{u\sqrt{u^2-1}} = \boxed{\sec^{-1}(3x)+C}$

4. $\int \frac{e^x}{1+e^{2x}}dx$

$u=e^x$
$du=e^x dx$

$\int \frac{1}{1+u^2}\,du$

$\boxed{\tan^{-1}(e^x)+C}$

5. $\int_{\ln 2}^{\ln(2/\sqrt{3})} \frac{e^{-x}dx}{\sqrt{1-e^{-2x}}}$

$u=e^{-x}$
$du=-e^{-x}dx$

$-\int_{1/2}^{\sqrt{3}/2} \frac{1}{\sqrt{1-u^2}}\,du$

$-\sin^{-1}(u)\Big]_{1/2}^{\sqrt{3}/2} = -\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)+\sin^{-1}\left(\frac{1}{2}\right)$

$= -\pi/3+\pi/6$

$= \boxed{-\pi/6}$

6. $\int_1^3 \frac{dx}{\sqrt{x}(x+1)}$

$u=\sqrt{x}$
$du=\frac{1}{2}x^{-1/2}dx$

$2\int \frac{du}{u^2+1}$

$\boxed{2\tan^{-1}(\sqrt{x})+C}$

7. $\int \frac{t}{t^4+1}dt$

$u=t^2$
$du=2t\,dt$

$\frac{1}{2}\int \frac{du}{u^2+1}$

$\boxed{\frac{1}{2}\tan^{-1}(t^2)+C}$

8. $\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}dx$

$u=\tan x$
$du=\sec^2 x\,dx$

$\int \frac{du}{\sqrt{1-u^2}}$

$\boxed{\sin^{-1}(\tan x)+C}$

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9. $\int \frac{\sin\theta}{\cos^2\theta+1}d\theta$

$u=\cos\theta$
$du=-\sin\theta\,d\theta$

$-\int \frac{1}{u^2+1}\,du$

$-\tan^{-1}(\cos\theta)+C$

10. $\int \frac{dx}{x\sqrt{1-(\ln x)^2}}$

$u=\ln x$
$du=1/x\,dx$

$\int \frac{1}{\sqrt{1-u^2}}\,du$

$\sin^{-1}(\ln x)+C$

11. $\int \frac{dx}{\sqrt{9-x^2}} =$

$u=\frac{1}{3}x$
$du=1/3\,dx$

$\frac{1}{3}\int \frac{dx}{\sqrt{9(1-\frac{1}{9}x^2)}}$

$\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(\frac{1}{3}x)+C$

12. $\int \frac{dx}{5+x^2} = \int \frac{dx}{5(1+\frac{1}{5}x^2)}$

$u=\frac{1}{\sqrt{5}}x$
$du=1/\sqrt{5}\,dx$

$\frac{\sqrt{5}}{5}\int \frac{du}{1+u^2} = \frac{\sqrt{5}}{5}\tan^{-1}(\frac{x}{\sqrt{5}})+C$

13. $\int \frac{dx}{x\sqrt{x^2-\pi}} = \frac{1}{\sqrt{\pi}}\int \frac{dx}{x\sqrt{\frac{x^2}{\pi}-1}}$

$u=\frac{x}{\sqrt{\pi}}$
$du=\frac{1}{\sqrt{\pi}}dx$

$\int \frac{du}{\sqrt{\pi}\cdot u\sqrt{u^2-1}}$

$\frac{1}{\sqrt{\pi}}\sec^{-1}(\frac{x}{\sqrt{\pi}})+C$

14. $\int \frac{e^x}{4+e^{2x}}dx = \frac{1}{4}\int \frac{e^x}{1+\frac{e^{2x}}{4}}dx$

$u=\frac{e^x}{2}$
$du=\frac{1}{2}e^x\,dx$

$\frac{1}{2}\int \frac{1}{1+u^2}\,du$

$\frac{1}{2}\tan^{-1}(\frac{e^x}{2})+C$

Use completing the square to help evaluate the following.

15. $\int \frac{dx}{x^2-2x+2}$

$x^2-2x+1-1+2$

$\int \frac{dx}{(x-1)^2+1}$

$u=x-1$
$du=dx$

$\int \frac{du}{u^2+1}$

$\tan^{-1}(x-1)+C$

16. $\int \frac{dx}{x^2+4x+13}$

$x^2+4x+4-4+13$

$\int \frac{dx}{(x+2)^2+9}$

$\frac{1}{9}\int \frac{dx}{\frac{(x+2)^2}{9}+1}$

$u=\frac{x+2}{3}$
$du=1/3\,dx$

$\frac{1}{3}\int \frac{1}{u^2+1}$

$\frac{1}{3}\tan^{-1}(\frac{x+2}{3})+C$

17. $\int \frac{dx}{\sqrt{-x^2-4x}}$

$-(x^2+4x+4-4)$
$-((x+2)^2-4)$
$4-(x+2)^2$

$\int \frac{dx}{\sqrt{4-(x+2)^2}}$

$\frac{1}{2}\int \frac{dx}{\sqrt{1-\frac{(x+2)^2}{4}}}$

$u=\frac{x+2}{2}$
$du=1/2\,dx$

$\int \frac{du}{\sqrt{1-u^2}}$

$\sin^{-1}(\frac{x+2}{2})+C$