Section	You should be able to ...	Examples	Review Exercises
7.5	1 Integrate a rational function whose denominator contains only distinct linear factors (p. 580)	2, 3	12, 27, 29
	2 Integrate a rational function whose denominator contains a repeated linear factor (p. 582)	4	17, 23
	3 Integrate a rational function whose denominator contains a distinct irreducible quadratic factor (p. 583)	5	2, 15
	4 Integrate a rational function whose denominator contains a repeated irreducible quadratic factor (p. 584)	6	22
	5 Work with the logistic model (p. 585)	7	53
7.6	1 Approximate an integral using the Trapezoidal Rule (p. 594)	1–5	49(a), 50
	2 Approximate an integral using trapezoidal sums (p. 599)	6	54
	3 Approximate an integral using Simpson's Rule (p. 600)	7–9	49(b)
7.7	1 Find integrals with an infinite limit of integration (p. 607)	1, 2	39, 42, 44
	2 Interpret an improper integral geometrically (p. 608)	3, 4	51, 52
	3 Integrate functions over $[a, b]$ that are not defined at an endpoint (p. 610)	5–7	40, 41, 43
	4 Use the Comparison Test for improper integrals (p. 612)	8	45, 46
7.8	1 Use a Table of Integrals (p. 616)	1–3	36(a)
	2 Use a computer algebra system (p. 618)	4	36(b)

REVIEW EXERCISES

In Problems 1–35, find each integral.

1. $\int \frac{dx}{x^2+4x+20}$
2. $\int \frac{y+1}{y^2+y+1}\,dy$
3. $\int \sec^3\phi \tan\phi\, d\phi$
4. $\int \cot^2\theta \csc\theta\, d\theta$
5. $\int \sin^3\phi\, d\phi$
6. $\int \frac{x^2}{\sqrt{4-x^2}}dx$
7. $\int \frac{dx}{\sqrt{(x+2)^2-1}}$
8. $\int_0^{\pi/4} x\sin(2x)\,dx$
9. $\int v\csc^2 v\, dv$
10. $\int \sin^2 x\cos^3 x\, dx$
11. $\int (4-x^2)^{3/2}\,dx$
12. $\int \frac{3x^2+1}{x^3+2x^2-3x}dx$
13. $\int \frac{e^{2t}\,dt}{e^t-2}$
14. $\int \frac{dy}{5+4y+4y^2}$
15. $\int \frac{x\,dx}{x^4-16}$
16. $\int x^3e^{x^2}\,dx$
17. $\int \frac{y^2\,dy}{(y+1)^3}$
18. $\int \frac{dx}{x^2\sqrt{x^2+25}}$
19. $\int x\sec^2 x\,dx$
20. $\int \frac{dx}{\sqrt{16+4x-2x^2}}$
21. $\int \ln(1-y)\,dy$
22. $\int \frac{x^3-2x-1}{(x^2+1)^2}dx$
23. $\int \frac{3x^2+2}{x^3-x^2}dx$
24. $\int \frac{dy}{\sqrt{2+3y^2}}$
25. $\int x^2\sin^{-1}x\,dx$
26. $\int \sqrt{16+9x^2}\,dx$
27. $\int \frac{dx}{x^2+2x}$
28. $\int \sin^4 y\cos^4 y\,dy$
29. $\int \frac{w-2}{1-w^2}dw$
30. $\int \frac{x}{\sqrt{x^2-4}}dx$
31. $\int \frac{1}{\sqrt{x}}\cos^2\sqrt{x}\,dx$
32. $\int \sin\left(\frac{\pi}{2}x\right)\sin(\pi x)\,dx$
33. $\int \sin x\cos(2x)\,dx$
34. $\int_0^1 \frac{x^2}{\sqrt{4-x^2}}dx$
35. $\int_0^{\sqrt{3}} \frac{x\,dx}{\sqrt{1+x^2}}$

36. (a) Find $\int \frac{\cos^2(2x)dx}{\sin^3(2x)}$ using a Table of Integrals.

(b) Find $\int \frac{\cos^2(2x)dx}{\sin^3(2x)}$ using a computer algebra system (CAS).

(c) Verify the results from (a) and (b) are equivalent.

In Problems 37 and 38, derive each formula where $n > 1$ is an integer.

37. $\int x^n \tan^{-1}x\,dx = \frac{x^{n+1}}{n+1}\tan^{-1}x - \frac{1}{n+1}\int \frac{x^{n+1}}{1+x^2}dx$

38. $\int x^n(ax+b)^{1/2}dx = \frac{2x^n(ax+b)^{3/2}}{(2n+3)a} - \frac{2bn}{(2n+3)a}\int x^{n-1}(ax+b)^{1/2}dx$

In Problems 39–42, determine whether each improper integral converges or diverges. If it converges, find its value.

39. $\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}dx$
40. $\int_0^1 \frac{\sin\sqrt{x}}{\sqrt{x}}dx$
41. $\int_0^1 \frac{x\,dx}{\sqrt{1-x^2}}$
42. $\int_{-\infty}^0 xe^x\,dx$

93. See Student Solutions Manual. **95.** See Student Solutions Manual. **97.** See Student Solutions Manual.
99. See Student Solutions Manual. **101.** $\frac{1}{s^2}$ **103.** $\frac{1}{1+s^2}$ **105.** $\frac{1}{s-a}$ **107.** 2 **109.** 1 **111.** See Student Solutions Manual.
113. $\frac{a+b}{2}$ **115.** $\sigma^2 = \frac{(b-a)^2}{12}, \sigma = \frac{b-a}{2\sqrt{3}}$

AP® Practice Problems

1. D **3.** D **5.** A **7.** $\frac{\pi}{3}$

Section 7.8

1. $\frac{e^{2x}}{5}(\sin x + 2\cos x) + C$ **3.** $\frac{(2+3x)^5}{9}\left(\frac{2+3x}{6} - \frac{2}{5}\right) + C$ **5.** $\frac{2}{15}(32 - 8x + 3x^2)\sqrt{6+3x} + C$ **7.** $\sqrt{x^2+4} - 2\ln\left|\frac{2+\sqrt{x^2+4}}{x}\right| + C$
9. $-\frac{x}{4\sqrt{x^2-4}} + C$ **11.** $\frac{x^4(\ln x)^2}{4} - \frac{x^4}{8}\left(\ln x - \frac{1}{4}\right) + C$ **13.** $\frac{x}{2}\sqrt{x^2-16} - 8\ln\left|x + \sqrt{x^2-16}\right| + C$
15. $\frac{x}{4}(6-x^2)^{3/2} + \frac{9}{4}x\sqrt{6-x^2} + \frac{27}{2}\sin^{-1}\frac{\sqrt{6}x}{6} + C$ **17.** $\frac{x-5}{2}\sqrt{10x-x^2} + \frac{25}{2}\cos^{-1}\frac{5-x}{5} + C$ **19.** $\frac{\sin(11x)}{22} + \frac{\sin(5x)}{10} + C$
21. $\frac{x^2+1}{2}\tan^{-1}x - \frac{x}{2} + C$ **23.** $\frac{x^5}{5}\left(\ln x - \frac{1}{5}\right) + C$ **25.** $\frac{\sinh(2x)}{4} - \frac{x}{2} + C$ **27.** $-\frac{\sqrt{8x-x^2}}{12x^2} - \frac{\sqrt{8x-x^2}}{48x} + C$
29. $\frac{1}{8}\left[\frac{x}{4+x^2} + \frac{1}{2}\tan^{-1}\frac{x}{2}\right] + C$ **31.** $x\cosh x - \sinh x + C$ **33.** $\frac{1}{6}(4x+5)^{3/2}\left(\frac{3}{5}x + \frac{1}{2}\right) + C$ **35.** $-\frac{9\sqrt{2}}{4} + \frac{135}{8}\cos^{-1}\frac{2\sqrt{2}}{3} + C$
37. $\frac{e^{2x}}{5}(\sin x + 2\cos x)$ **39.** $\frac{27}{2}x^6 + \frac{216}{5}x^5 + 54x^4 + 32x^3 + 8x^2$ **41.** $\frac{2}{45}\sqrt{3}\sqrt{x+2}\,(3x^2 - 8x + 32)$ **43.** $\sqrt{x^2+4} - 2\operatorname{arctanh}\frac{2}{\sqrt{x^2+4}}$
45. $-\frac{1}{4}\frac{x}{\sqrt{x^2-4}}$ **47.** $\frac{1}{32}x^4(8\ln^2 x - 4\ln x + 1)$ **49.** $\frac{1}{2}x\sqrt{x^2-16} - 8\ln(x + \sqrt{x^2-16})$
51. $\frac{9}{4}x\sqrt{6-x^2} - \frac{27}{2}i\ln\left(ix + \sqrt{6-x^2}\right) + \frac{1}{4}x(6-x^2)^{3/2}$ **53.** $\frac{1}{2}x\sqrt{-x(x-10)} - \frac{25}{2}i\ln(ix + \sqrt{-x(x-10)} - 5i) - \frac{5}{2}\sqrt{-x(x-10)}$
55. $\frac{1}{10}\sin 5x + \frac{1}{22}\sin 11x$ **57.** $\frac{1}{2}\arctan x - \frac{1}{2}x - \frac{1}{4}\pi + \frac{1}{2}x^2\arctan x$ **59.** $\frac{1}{5}x^5\ln x - \frac{1}{25}x^5$ **61.** $-\frac{1}{8e^{2x}}(-e^{2(2x)} + 4xe^{2x} + 1)$
63. $\frac{x\left(8x + 8\sqrt{8x-1}\,x\tan^{-1}\sqrt{8x-1} - 1\right)}{\sqrt{x^4(8x-1)}}$ **65.** $\frac{1}{32(x^2+4)}\left(4x - 4\pi + 8\arctan\frac{1}{2}x + 2x^2\arctan\frac{1}{2}x - \pi x^2\right)$
67. $\frac{1}{2e^{-x}}(x + e^{2(-x)} + xe^{2(-x)} - 1)$ **69.** $\frac{1}{60}(4x+5)^{3/2}(6x+5)$ **71.** $-\frac{9\sqrt{2}}{4} + \frac{135}{8}\cos^{-1}\frac{2\sqrt{2}}{3}$ **73.** Yes **75.** No **77.** No

Review Exercises

1. $\frac{1}{4}\tan^{-1}\frac{x+2}{4} + C$ **3.** $\frac{1}{3}\sec^3\phi + C$ **5.** $\frac{1}{3}\cos^3\phi - \cos\phi + C$ **7.** $\ln\left|x + 2 + \sqrt{(x+2)^2 - 1}\right| + C$ **9.** $-v\cot v + \ln|\sin v| + C$
11. $6\sin^{-1}\frac{x}{2} + 2x\sqrt{4-x^2} + \frac{1}{4}x\sqrt{4-x^2}\,(2-x^2) + C$ **13.** $e^t + 2\ln\left|e^t - 2\right| + C$ **15.** $\frac{1}{16}\ln\left|\frac{x^2-4}{x^2+4}\right| + C$
17. $\ln|y+1| + \frac{2}{y+1} - \frac{1}{2(y+1)^2} + C$ **19.** $x\tan x - \ln|\sec x| + C$ **21.** $y\ln|1-y| - y - \ln|1-y| + C$ **23.** $-2\ln|x| + \frac{2}{x} + 5\ln|x-1| + C$
25. $\frac{1}{3}x^3\sin^{-1}x + \frac{1}{3}\sqrt{1-x^2} - \frac{1}{9}\left(1-x^2\right)^{3/2} + C$ **27.** $\frac{1}{2}\ln\left|\frac{x}{x+2}\right| + C$ **29.** $\frac{1}{2}\ln|1-w| - \frac{3}{2}\ln|w+1| + C$
31. $\sqrt{x} + \sin\sqrt{x}\cos\sqrt{x} + C$ **33.** $\frac{1}{2}\cos x - \frac{1}{6}\cos(3x) + C$ **35.** 1 **37.** See Student Solutions Manual. **39.** Converges to $\frac{2}{e}$.
41. Converges to 1. **43.** See Student Solutions Manual. **45.** Diverges **47.** $f(x) = x^2\sin x$ **49.** (a) 1.910 (b) 1.910 **51.** 3
53. (a) $M = 100$ (b) 0.24 or 24% (c) 50

Chapter 7 AP® Review Problems

1. B **3.** C **5.** D **7.** D **9.** D **11.** C

AP® Practice Exam: Calculus AB

Section 1: Multiple Choice:

1. D **2.** C **3.** A **4.** B **5.** B **6.** C **7.** B **8.** C **9.** D **10.** B **11.** B **12.** C **13.** D **14.** B **15.** D **16.** B **17.** C **18.** B
19. C **20.** D **21.** A **22.** B **23.** D **24.** B **25.** A **26.** C **27.** D **28.** B **29.** B **30.** D **31.** B **32.** D **33.** B **34.** A **35.** B
36. C **37.** A **38.** B **39.** C **40.** B **41.** B **42.** C **43.** C **44.** D **45.** B

① $\int \frac{dx}{x^2+4x+20}$ $\quad x+2 = 4\tan\theta$, $dx = 4\sec^2\theta\, d\theta$

$\int \frac{dx}{x^2+4x+4+20-4}$

$\int \frac{dx}{(x+2)^2+16}$

$\int \frac{4\sec^2\theta\, d\theta}{16\tan^2\theta+16}$

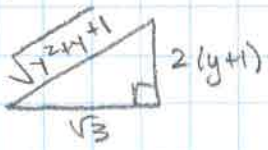
$\int \frac{1}{4}\, d\theta$

$\frac{1}{4}\theta + C$

$\frac{1}{4}\tan^{-1}\left(\frac{x+2}{4}\right) + C$

② $\int \frac{(y+1)}{y^2+y+\frac{1}{4}+1-\frac{1}{4}}\, dy$ $\quad y+\frac{1}{2} = \frac{\sqrt{3}}{2}\tan\theta$, $dy = \frac{\sqrt{3}}{2}\sec^2\theta\, d\theta$

$\int \frac{y+1}{(y+1/2)^2+3/4}\, dy$



$\int \frac{\frac{\sqrt{3}}{2}\tan\theta+\frac{1}{2}}{\frac{3}{4}\tan^2\theta+\frac{3}{4}} \frac{\sqrt{3}}{2}\sec^2\theta\, d\theta$

$\frac{2\sqrt{3}}{3}\int \frac{\sqrt{3}}{2}\tan\theta + 1/2$

$\frac{2\sqrt{3}}{3}\cdot\left(-\frac{\sqrt{3}}{2}\ln|\cos\theta| + \frac{1}{2}\theta\right) + C$

$-\ln\left|\frac{\sqrt{3}}{\sqrt{y^2+y+1}}\right| + \frac{\sqrt{3}}{3}\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) + C$

⑤ $\int \sin^3\phi\, d\theta$ $\quad u = \cos\theta$, $du = -\sin\theta\, d\theta$

$\int \sin^2\phi \sin\phi\, d\phi$

$\int (1-\cos^2\phi)\sin\phi\, d\phi$

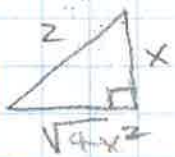
$-\int (1-u^2)\, d\theta$

$-u + \frac{1}{3}u^3 + C$

$-\cos\phi + \frac{1}{3}\cos^3\phi + C$

⑥ $\int \frac{x^2}{\sqrt{4-x^2}}\, dx$ $\quad x = 2\sin\theta$, $dx = 2\cos\theta\, d\theta$

$\int \frac{4\sin^2\theta \cdot 2\cos\theta\, d\theta}{\sqrt{4-4\sin^2\theta}}$



$4\int \cdot \frac{1}{2}(1-\cos 2\theta)\, d\theta$

$2(\theta - \frac{1}{2}\sin 2\theta) + C$

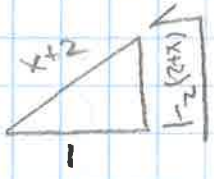
$2\theta - 2\sin\theta\cos\theta + C$

$2\sin^{-1}\left(\frac{x}{2}\right) - 2\cdot\frac{x}{2}\cdot\frac{\sqrt{4-x^2}}{2} + C$

$2\sin^{-1}\left(\frac{x}{2}\right) - \frac{x\sqrt{4-x^2}}{2} + C$

(7) $\int \frac{dx}{\sqrt{(x+2)^2-1}}$ $\quad x+2=\sec\theta$ $\quad dx=\sec\theta\tan\theta\,d\theta$

$\int \frac{\sec\theta\tan\theta\,d\theta}{\sqrt{\sec^2\theta-1}}$



$\ln|\sec\theta+\tan\theta|+C$

$\ln\left|(x+2)+\sqrt{(x+2)^2-1}\right|+C$

(10) $\int \sin^2x\cos^2x\cdot\cos x\,dx$

$\int \sin^2x(1-\sin^2x)\cos x\,dx$

$\int u^2(1-u^2)\,du$ $\quad u=\sin x$ $\quad du=\cos x$

$\int u^2-u^4\,du$

$\frac{1}{3}u^3-\frac{1}{5}u^5+C$

$\frac{1}{3}\sin^3x-\frac{1}{5}\sin^5x+C$

(11) $\int (4-x^2)^{3/2}\,dx$ $\quad x=2\sin\theta$ $\quad dx=2\cos\theta\,d\theta$

$\int(4-4\sin^2\theta)^{3/2}\cdot 2\cos\theta\,d\theta$

$16\int \cos^3\theta\cdot\cos\theta\,d\theta$

$16\int \cos^4\theta\,d\theta$

$16\cdot\int\frac{1}{4}(1+\cos2\theta)^2\,d\theta$

$4\int 1+2\cos2\theta+\cos^2 2\theta\,d\theta$

$4\int 1+2\cos2\theta+\frac{1}{2}(1+\cos4\theta)\,d\theta$

(14) $\int \frac{dy}{5+1+4(y^2+y+1/4)}$

$\int \frac{dy}{4+4(y+1/2)^2}$ $\quad y+1/2=\tan\theta$ $\quad dy=\sec^2\theta\,d\theta$

$\int \frac{\sec^2\theta\,d\theta}{4+4\tan^2\theta}$

$\frac{1}{4}\int d\theta$

$\frac{1}{4}\theta+C$

$\frac{1}{4}\tan^{-1}(y+1/2)+C$

$4\int \frac{3}{2}+2\cos2\theta+\frac{1}{2}\cos4\theta\,d\theta$

$4\left(\frac{3}{2}\theta+\sin2\theta+\frac{1}{8}\sin4\theta\right)d\theta$

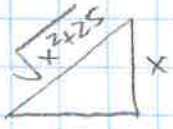
$6\theta+8\sin\theta\cos\theta+\frac{1}{2}\cdot 2\sin2\theta\cos2\theta$

$6\theta+8\sin\theta\cos\theta+\sin\theta\cos\theta(\cos^2\theta-\sin^2\theta)$

$6\sin^{-1}\left(\frac{x}{2}\right)+8\cdot\frac{x}{2}\cdot\frac{\sqrt{4-x^2}}{2}+\frac{x}{2}\cdot\frac{\sqrt{4-x^2}}{2}\left(\frac{4-x^2}{4}-\frac{x^2}{4}\right)+C$

(18) $\int \frac{dx}{x^2\sqrt{x^2+25}}$ $\quad x = 5\tan\theta$, $dx = 5\sec^2\theta\, d\theta$

$$\int \frac{5\sec^2\theta\, d\theta}{25\tan^2\theta\sqrt{25+25\tan^2\theta}}$$



$$\frac{1}{25}\int \cot^2\theta\, d\theta$$

$$\frac{1}{25}\int \csc^2\theta - 1\, d\theta$$

$$\frac{1}{25}\left(-\csc\theta\cot\theta - \theta\right) + C$$

$$-\frac{1}{25}\left(\frac{\sqrt{x^2+25}}{x}\cdot\frac{5}{x} - \tan^{-1}\frac{x}{5}\right) + C$$

(20) $\int \frac{dx}{\sqrt{16+2-2(x^2-2x+1)}}$

$$\int \frac{dx}{\sqrt{18-2(x-1)^2}}$$

$x-1 = 3\sin\theta$, $dx = 3\cos\theta\, d\theta$

$$\frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{9-(x-1)^2}}$$

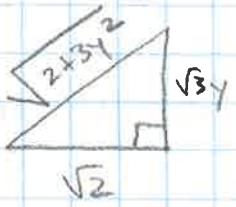
$$\frac{1}{\sqrt{2}}\int \frac{3\cos\theta\, d\theta}{3\sqrt{1-\sin^2\theta}}$$

$$\frac{1}{\sqrt{2}}\,\theta + C$$

$$\frac{1}{\sqrt{2}}\cdot\sin^{-1}\left(\frac{x-1}{3}\right) + C$$

(24) $\int \frac{dy}{\sqrt{2+3y^2}}$ $\quad y = \sqrt{\frac{2}{3}}\tan\theta$, $dy = \sqrt{\frac{2}{3}}\sec^2\theta\, d\theta$

$$\int \frac{\sqrt{\frac{2}{3}}\sec^2\theta\, d\theta}{\sqrt{2+2\tan^2\theta}}$$



$$= \frac{1}{\sqrt{3}}\int \sec\theta\, d\theta$$

$$= \frac{1}{\sqrt{3}}\ln\left|\sec\theta + \tan\theta\right| + C$$

$$= \frac{1}{\sqrt{3}}\ln\left|\frac{\sqrt{2+3y^2}}{\sqrt{2}} + \frac{\sqrt{3}y}{\sqrt{2}}\right| + C$$

(26) $\int \sqrt{16+9x^2}\, dx$ $\quad x = \frac{4}{3}\tan\theta$, $dx = \frac{4}{3}\sec^2\theta\, d\theta$

$$\int \sqrt{16+16\tan^2\theta}\cdot\frac{4}{3}\sec^2\theta\, d\theta$$

$$\frac{16}{3}\int \sec^3\theta\, d\theta$$

(28) $\int \sin^4 y \cos^4 y \, dy$

$\int \left(\frac{1}{2}(1-\cos 2\theta)\right)^2 \left(\frac{1}{2}(1+\cos 2\theta)\right)^2 dy$

$\frac{1}{16}\int \left[(1-\cos 2\theta)(1+\cos 2\theta)\right]^2 d\theta$

$\frac{1}{16}\int (1-\cos^2 2\theta)^2 d\theta$

$\frac{1}{16}\int \sin^4 2\theta \, d\theta$

$\frac{1}{16}\int (1-\cos 4\theta)^2 d\theta$

$\frac{1}{16}\int 1-2\cos 4\theta + \cos^2 4\theta \, d\theta$

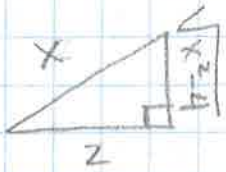
$\frac{1}{16}\int 1-2\cos 4\theta + \frac{1}{2}(1+\cos 8\theta) \, d\theta$

$\frac{1}{16}\int \frac{3}{2} - 2\cos 4\theta + \frac{1}{2}\cos 8\theta \, d\theta$

$\frac{1}{16}\left(\frac{3}{2}\theta - \frac{1}{2}\sin 4\theta + \frac{1}{16}\sin 8\theta\right) + C$

(30) $\int \frac{x}{\sqrt{x^2-4}} dx$ $\quad x = 2\sec\theta$, $dx = 2\sec\theta\tan\theta \, d\theta$

$\int \frac{2\sec\theta \cdot 2\sec\theta\tan\theta \, d\theta}{\sqrt{4\sec^2\theta - 4}}$



$2\int \sec^2\theta \, d\theta$

$2\tan\theta + C$

$2\cdot\frac{\sqrt{x^2-4}}{x} + C$

(33) $\int \sin x \cdot (2\cos^2 x - 1) \, dx$

$u = \cos x$, $du = -\sin x$

$-\int 2u^2 - 1 \, du$

$-\frac{2}{3}u^3 + u + C$

$-\frac{2}{3}\cos^3 x + \cos x + C$

(34) $\int_0^1 \frac{x^2}{\sqrt{4-x^2}} dx$ $\quad x = 2\sin\theta$, $dx = 2\cos\theta$

$\int_0^{\pi/6} \frac{4\sin^2\theta \cdot 2\cos\theta}{\sqrt{4-4\sin^2\theta}}$

$4\int_0^{\pi/6} \sin^2\theta \, d\theta$

$2\int_0^{\pi/6} 1-\cos 2\theta \, d\theta$

$2\left(\theta - \frac{1}{2}\sin 2\theta\right)\Big|_0^{\pi/6}$

$2\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) = \boxed{\frac{\pi}{3} - \frac{\sqrt{3}}{2}}$

(41) $\int_0^1 \frac{x\,dx}{\sqrt{1-x^2}}$ $\quad x = \sin\theta$, $dx = \cos\theta\,d\theta$

$$\int_0^{\pi/2} \frac{\sin\theta \cdot \cos\theta\,d\theta}{\sqrt{1-\sin^2\theta}}$$

$$-\cos\theta\Big|_0^{\pi/2} = -\cos\tfrac{\pi}{2} + \cos 0$$

$$= 1$$

∴ CONVERGES