Recall...

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + C \qquad 1+\tan^2\theta = \sec^2\theta$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C \qquad 1-\sin^2\theta = \cos^2\theta$$

$$\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}x + C \qquad \sec^2\theta - 1 = \tan^2\theta$$

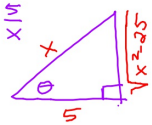
Page 1

$$\int \frac{\sqrt{x^2-25}}{x}\,dx$$

TRIG SUBSTITUTION:

$x = 5\sec\theta \Rightarrow \sec\theta = \frac{x}{5} \Rightarrow \cos\theta = \frac{5}{x}$

$dx = 5\sec\theta\tan\theta\,d\theta$



$$\int \frac{\sqrt{25\sec^2\theta - 25}}{5\sec\theta}\cdot 5\sec\theta\tan\theta\,d\theta$$

$$\int \frac{5\sqrt{\sec^2\theta - 1}}{1}\cdot\tan\theta\,d\theta$$

$$\int 5\sqrt{\tan^2\theta}\cdot\tan\theta\,d\theta$$

$$5\int \tan^2\theta\,d\theta$$

$$5\int \sec^2\theta - 1\,d\theta$$

$$5(\tan\theta - \theta) + C$$

$$5\left(\frac{\sqrt{x^2-25}}{5} - \sec^{-1}\left(\frac{x}{5}\right)\right) + C$$

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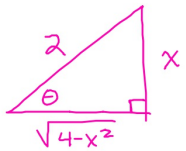
$$\int \frac{dx}{x^2\sqrt{4-x^2}} \qquad x = 2\sin\theta \Rightarrow \frac{x}{2} = \sin\theta$$

$dx = 2\cos\theta\,d\theta$



$$\int \frac{2\cos\theta\,d\theta}{4\sin^2\theta\cdot\sqrt{4-4\sin^2\theta}}$$

$\sqrt{4(1-\sin^2\theta)}$

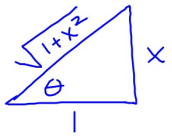
$$\frac{1}{4}\int \csc^2\theta\,d\theta$$

$$-\frac{1}{4}\cot\theta + C = -\frac{1}{4}\left(\frac{\sqrt{4-x^2}}{x}\right) + C$$

Page 3

$$\int \frac{dx}{(1+x^2)^{3/2}} \qquad x = \tan\theta \Rightarrow$$

$dx = \sec^2\theta\,d\theta$



$$\int \frac{\sec^2\theta\,d\theta}{(1+\tan^2\theta)^{3/2}}$$

$\sec^2\theta$

$$\int \frac{\sec^2\theta}{\sec^3\theta}\,d\theta$$

$$\int \cos\theta\,d\theta = \sin\theta + C = \frac{x}{\sqrt{1+x^2}} + C$$

Page 4

IF INTEGRAND HAS . . .	USE THE SUBSTITUTION . . .
a^2+x^2	$x = a\tan\theta$
$a^2+(bx)^2$	$x = \frac{a}{b}\tan\theta$
a^2-x^2	$x = a\sin\theta$
$a^2-(bx)^2$	$x = \frac{a}{b}\sin\theta$
x^2-a^2	$x = a\sec\theta$
$(bx)^2-a^2$	$x = \frac{a}{b}\sec\theta$

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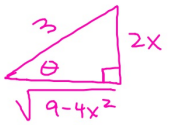
$$\int \frac{x^2}{\sqrt{9-4x^2}}\,dx$$

$x = \frac{3}{2}\sin\theta \Rightarrow \frac{2x}{3} = \sin\theta$

$dx = \frac{3}{2}\cos\theta\,d\theta$



$$\int \frac{\frac{9}{4}\sin^2\theta}{\sqrt{9-4\cdot\frac{9}{4}\sin^2\theta}}\cdot\frac{3}{2}\cos\theta\,d\theta$$

$$\sqrt{1-\sin^2\theta}$$

$$\frac{9}{8}\int \sin^2\theta\,d\theta$$

$$\frac{9}{16}\int 1-\cos 2\theta\,d\theta$$

$$\frac{9}{16}\left(\theta - \frac{1}{2}\sin 2\theta\right) + C$$

★ $\sin 2\theta = 2\sin\theta\cos\theta$ ★ MEM-O-RIZE ★

$$\frac{9}{16}\left(\theta - \sin\theta\cos\theta\right) + C$$

$$\frac{9}{16}\left(\sin^{-1}\left(\frac{2x}{3}\right) - \frac{2x}{3}\cdot\frac{\sqrt{9-4x^2}}{3}\right) + C$$

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OTHER NOTES

$$\int \sec\theta\,d\theta = \ln|\sec\theta+\tan\theta| + C$$

$$\int \csc\theta\,d\theta = -\ln|\csc\theta+\cot\theta| + C$$

$$\int \tan\theta\,d\theta = \boxed{-\ln|\cos\theta|} + C = \ln|\sec\theta| + C$$

$$\int \cot\theta\,d\theta = \boxed{\ln|\sin\theta|} + C = -\ln|\csc\theta| + C$$

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