

Partial Fractions

Section 8.4

A basic example:

$$\int \frac{2}{x-4} + \frac{3}{x+1} dx = 2 \ln|x-4| + 3 \ln|x+1|$$

In general:

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln|ax+b| + C$$



More complicated:

$$\int \frac{5x-10}{x^2-3x-4} dx = \int \frac{5x-10}{(x-4)(x+1)} dx = \int \frac{\cancel{A}^2}{x-4} + \frac{\cancel{B}^3}{x+1} dx$$

$$A(x+1) + B(x-4) = 5x-10$$

$$x=4: A(5) = 10 \Rightarrow \boxed{A=2}$$

$$x=-1: B(-5) = -15 \Rightarrow \boxed{B=3}$$

$$= \boxed{2 \ln|x-4| + 3 \ln|x+1| + C}$$



Partial Fraction Decomposition Criteria:

1. DEGREE DENOM. $>$ DEGREE NUMERATOR.

2. DENOMINATOR MUST BE FACTORABLE.



Example:

$$\int \frac{dx}{x^2 - x - 2} = \int \frac{dx}{(x-2)(x+1)} = \int \frac{A^{-1/3}}{x-2} + \frac{B^{-1/3}}{x+1} dx$$

$$A(x+1) + B(x-2) = 1$$

$$x = -1: B(-3) = 1 \Rightarrow \boxed{B = -1/3}$$

$$x = 2: A(3) = 1 \Rightarrow \boxed{A = 1/3}$$

$$= \boxed{\frac{1}{3} \ln|x-2| - \frac{1}{3} \ln|x+1| + C}$$

$$= \boxed{\frac{1}{3} \ln \left| \frac{x-2}{x+1} \right| + C}$$



Example: Repeating Linear Factor

$$\int \frac{2x+4}{x^3-2x^2} dx = \int \frac{2x+4}{x^2(x-2)} dx = \int \frac{\cancel{A}^{-2}}{x} + \frac{\cancel{B}^{-2}}{x^2} + \frac{\cancel{C}^2}{x-2} dx$$

$$Ax(x-2) + B(x-2) + Cx^2 = 2x+4$$

$$x=0: B(-2)=4 \Rightarrow \boxed{B=-2}$$

$$x=2: C(4)=8 \Rightarrow \boxed{C=2}$$

$$x=3: A(3)(1) + -2(1) + 2(9) = 10$$

$$3A + 16 = 10$$

$$3A = -6$$

$$\boxed{A=-2}$$

$$-2 \ln|x| + \frac{2}{x} + 2 \ln|x-2| + C$$

$$2 \ln \left| \frac{x-2}{x} \right| + \frac{2}{x} + C$$



Example: Use long division to help

$$\int \frac{3x^3 + x}{x-2} dx = \int 3x^2 + 6x + 13 + \frac{26}{x-2} dx$$

$$\begin{array}{r} 3x^2 + 6x + 13 \\ x-2 \overline{) 3x^3 + 0x^2 + x + 0} \\ \underline{-(3x^3 - 6x^2)} \\ 6x^2 + x + 0 \\ \underline{-(6x^2 - 12x)} \\ 13x + 0 \\ \underline{-(13x - 26)} \\ 26 \end{array}$$

$$x^3 + 3x^2 + 13x + 26 \ln|x-2| + C$$



Example: Use long division to help

$$\int \frac{3x^4 + 3x^3 - 5x^2 + x - 1}{x^2 + x - 2} dx = \int 3x^2 + 1 + \frac{1}{(x+2)(x-1)} dx$$

$$\begin{array}{r} 3x^2 + 1 \\ x^2 + x - 2 \overline{) 3x^4 + 3x^3 - 5x^2 + x - 1} \\ \underline{-(3x^4 + 3x^3 - 6x^2)} \\ x^2 + x - 1 \\ \underline{-(x^2 + x - 2)} \\ 1 \end{array}$$

$$\int 3x^2 + 1 + \frac{-1/3}{x+2} + \frac{1/3}{x-1} dx$$

$$x^3 + x - \frac{1}{3} \ln|x+2| + \frac{1}{3} \ln|x-1| + C$$



Homework:

Section 8.4 WS

