Integration by Parts

Section 6.3

Recall the product rule:

$$(uv)' = uv' + vu'$$

$$\int (uv)' = \int u\,dv + \int v\,du$$

$$uv = \int u\,dv + \int v\,du$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Examples:

$$\int \underbrace{x}_{u}\underbrace{e^x dx}_{dv}$$

$u = x \qquad dv = e^x\,dx$

$du = dx \qquad v = e^x$

$$xe^x - \int e^x\,dx$$

$$\boxed{xe^x - e^x + C}$$

CHECK:

$$xe^x + e^x \cdot (1) - e^x$$

$$xe^x + e^x - e^x$$

✓

Examples:

$$\int \ln x\,dx$$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}\,dx \qquad v = x$

$$x\ln x - \int 1\,dx$$

$$\boxed{x\ln x - x + C}$$

Examples:

$$\int_0^1 \tan^{-1} x\,dx$$

$u = \tan^{-1}x \qquad dv = dx$

$du = \frac{1}{1+x^2}\,dx \qquad v = x$

$x\tan^{-1}x - \int \frac{x}{1+x^2}\,dx$

$u = 1+x^2$

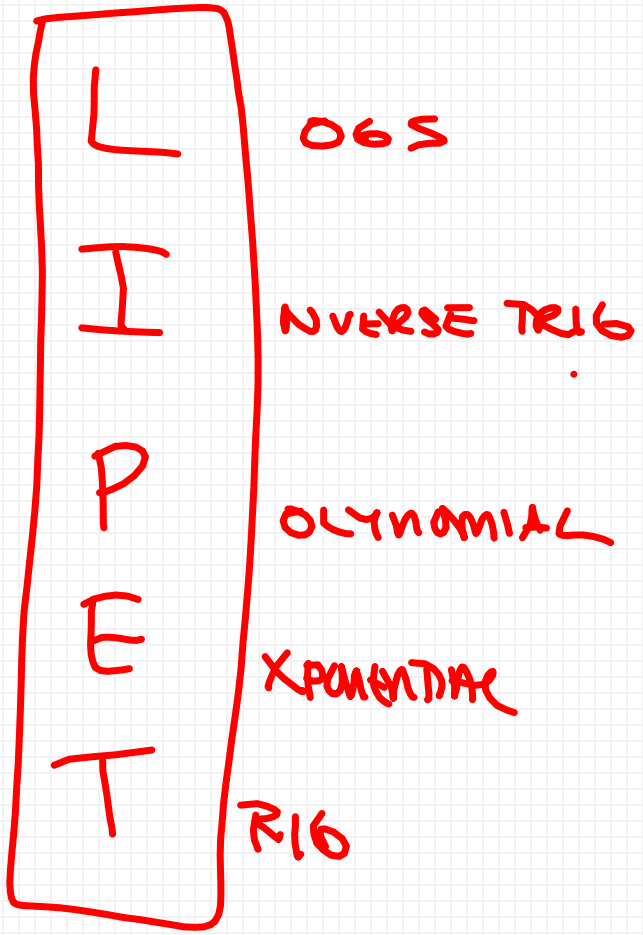
$du = 2x\,dx$

$x\tan^{-1}x - \frac{1}{2}\int \frac{1}{u}\,du$

$x\tan^{-1}x - \frac{1}{2}\ln|1+x^2|\Big]_0^1 = \tan^{-1}(1) - \frac{1}{2}\ln|2| - 0\tan^{-1}(0) + \frac{1}{2}\ln|1|$

$= \boxed{\frac{\pi}{4} - \frac{1}{2}\ln 2}$

How do you know what to pick for *u*?



$$\int \ln x \cdot e^x \, dx \qquad u = \ln x$$

$$\int \sin^{-1}(x)\left(3x^2+1\right) dx \qquad u = \sin^{-1} x$$

Tabular Integration: works for PE or PT problems

$$\int x^2 e^{-x}\,dx$$

$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$

$= -e^{-x}(x^2+2x+2) + C$

	u	dv
+	x^2	e^{-x}
−	$2x$	$-e^{-x}$
+	2	e^{-x}
−	0	$-e^{-x}$

Modified Table: works for ET problems only

$$2\int e^x \cos x\,dx = e^x\sin x + e^x\cos x - \int e^x\cos x\,dx$$

	u	dv
+	e^x	$\cos x$
−	e^x	$\sin x$
+	e^x	$-\cos x$

$$2\int e^x\cos x\,dx = \frac{e^x\sin x + e^x\cos x}{2} + C$$

Modified Table: works for ET problems only

$$\int e^{2x}\cos 3x\,dx = \frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x - \frac{4}{9}\int e^{x}\cos 3x\,dx$$

	u	dv
+	e^{2x}	$\cos 3x$
−	$2e^{2x}$	$\frac{1}{3}\sin 3x$
+	$4e^{2x}$	$-\frac{1}{9}\cos 3x$

$$\frac{13}{9}\int e^{2x}\cos 3x\,dx = \frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x$$

$$\int e^{2x}\cos 3x\,dx = \frac{9}{13}\left(\frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x\right) + C$$

Numerical Example:

$$\int_0^3 x^2 f''(x)\,dx$$

x	0	1	2	3
$f(x)$	5	2	3	6
$f'(x)$	−3	1	3	4

	u	dv
+	x^2	$f''(x)$
−	$2x$	$f'(x)$
+	2	$f(x)$
−	0	$\int f(x)\,dx$

$$\left(x^2 f'(x) - 2x f(x)\right]_0^3 + 2\int_0^3 f(x)\,dx$$

APPROXIMATE w/ TRAP.

$3^2 f'(3) - 2(3) f(3)$ + $2\left[\frac{1}{2}(1)\left[5 + 2(2) + 2(3) + 6\right]\right]$

$9(4) - 6(6)$ + 21

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Homework:

Day 1: Section 6.3 WS - odds

Day 2: AP Packet #54-59