Differential Equations

Section 6.1

Differential Equation: AN EQUATION W/ DERIVATIVE(S)

Example: Show that the given function is a solution to the given equation.

$$y = e^{2x}$$

$y' = 2e^{2x}$

$y'' = 4e^{2x}$

$$y'' - 3y' + 2y = 0$$

$4e^{2x} - 3 \cdot 2e^{2x} + 2 \cdot e^{2x} \stackrel{?}{=} 0$

$0 = 0$ ✓

Solve the differential equation: Solve for .... y

$$\frac{dy}{dx} = -4xy^2 \qquad f(0) = 1$$

INITIAL CONDITION

x y

$$\int \frac{1}{y^2}\,dy = \int -4x\,dx$$

$$-\frac{1}{y} = -2x^2 + C_1$$

$$\frac{1}{y} = 2x^2 + C_2$$

$$\boxed{y = \frac{1}{2x^2 + C_2}}$$

GENERAL SOLUTION

SOLVING FOR C LATE.

$$1 = \frac{1}{2(0)^2 + C}$$

$$C = 1$$

$$\boxed{y = \frac{1}{2x^2+1}}$$

PARTICULAR SOLUTION

SOLVE FOR C EARLY.

$$-\frac{1}{y} = -2x^2 + C$$

$$-\frac{1}{1} = -2(0)^2 + C$$

$$-1 = C$$

$$-\frac{1}{y} = -2x^2 - 1$$

$$\boxed{y = \frac{1}{2x^2+1}}$$

→

Solve the differential equation: Solve for ....

$$x\left(y-1\right)\frac{dy}{dx}=y$$

$$\boxed{y=\ln|x\cdot y|+C}$$

$$\frac{y-1}{y}\,dy=\frac{1}{x}\,dx$$

$$1-\frac{1}{y}\,dy=\frac{1}{x}\,dx$$

$$y-\ln|y|=\ln|x|+C$$

$$y=\ln|y|+\ln|x|+C$$

Solve the differential equation: Solve for ….

Solve LATE IF you e BOTH SIDES.

$$\frac{dy}{dx} = yx^2 \quad y(0) = -3$$

$\frac{1}{y}\,dy = x^2\,dx$

$\ln|y| = \frac{1}{3}x^3 + C$

SOLVE EARLY

$\ln|-3| = 0 + C$

$e^{\ln|y|} = e^{\frac{1}{3}x^3 + \ln 3}$

$|y| = e^{1/3 x^3} \cdot e^{\ln 3}$

$|y| = 3e^{1/3 x^3}$

$\boxed{y = -3e^{1/3 x^3}}$

SOLVE LATE ★

$e^{\ln|y|} = e^{1/3 x^3 + C}$

$|y| = e^{\frac{1}{3}x^3} \cdot e^{C} \rightarrow$ constant

$|y| = C_2 e^{1/3 x^3}$

$y = \pm C_2 e^{1/3 x^3}$

$y = C_3 e^{1/3 x^3}$

$-3 = Ce^0$

$-3 = C$

$\Rightarrow \boxed{y = -3e^{1/3 x^3}}$

Chapter 6 AP Packet #48 (1998 AB4):

Let *f* be a function with $f(1) = 4$ such that for all points on the graph of *f*, the slope is given by $\frac{3x^2+1}{2y}$. $= \frac{dy}{dx}$

a. Find the slope of the graph of *f* at the point where *x* = 1.

$$\left.\frac{dy}{dx}\right|_{(1,4)} = \frac{3(1)^2+1}{2(4)} = \frac{1}{2}$$

Chapter 6 AP Packet #48 (1998 AB4):

Let *f* be a function with $f(1) = 4$ such that for all points on the graph of *f*, the slope is given by $\frac{3x^2+1}{2y}$.

b. Write an equation for the line tangent to the graph of *f* at *x* = 1 and use it to approximate $f(1.2)$.

$$\boxed{y-4=\tfrac{1}{2}(x-1)}$$

$$y=4+\tfrac{1}{2}(x-1)$$

$$f(1.2)\approx 4+\tfrac{1}{2}(1.2-1) = \boxed{4.1}$$

Chapter 6 AP Packet #48 (1998 AB4):

c. Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2+1}{2y}$ with the initial condition $f(1) = 4$.

$2y\,dy = (3x^2+1)\,dx$

$y^2 = x^3 + x + C$

$4^2 = 1^3 + 1 + C$

$14 = C$

$\sqrt{y^2} = \sqrt{x^3+x+14}$

$|y| = \sqrt{x^3+x+14}$

$$\boxed{y = \pm\sqrt{x^3+x+14}}$$

KEEP $+\sqrt{\ }$ SINCE $f(1) = +4$

Chapter 6 AP Packet #48 (1998 AB4):

d. Use your solution from part c) to find $f(1.2)$.

$$f(1.2) = \sqrt{(1.2)^3 + (1.2) + 14} = 4.114$$

Homework:

Chapter 6 AP Packet #43 – 53 odd