Inverse Trig Integrals

Section 6.2

The basic ones:

$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}} \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$$

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2} \qquad \int \frac{1}{1+x^2}\,dx = \tan^{-1}x + C$$

$$\frac{d}{dx}\left(\sec^{-1} x\right) = \frac{1}{x\sqrt{x^2-1}} \qquad \int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}(x) + C$$

Examples:

$\sec\theta = \sqrt{2}$
$\cos\theta = \sqrt{2}/2$

$\sec\theta = 2$
$\cos\theta = 1/2$

$$\int_0^{\frac{\sqrt{2}}{2}} \frac{dx}{\sqrt{1-x^2}}$$

$\sin^{-1}(x)\Big]_0^{\sqrt{2}/2}$

$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) - \sin^{-1}(0)$

$\frac{\pi}{4} - 0$

$\pi/4$

$$\int_{-1}^{1} \frac{dx}{1+x^2}$$

$\tan^{-1}x\Big]_{-1}^{1}$

$\tan^{-1}(1) - \tan^{-1}(-1)$

$\frac{\pi}{4} + \frac{\pi}{4}$

$\pi/2$

$$\int_{\sqrt{2}}^{2} \frac{dx}{x\sqrt{x^2-1}}$$

$\sec^{-1}(x)\Big]_{\sqrt{2}}^{2}$

$\sec^{-1}(2) - \sec^{-1}(\sqrt{2})$

$\pi/3 - \pi/4$

$\pi/12$

Problems with u-substitution:

$$\int \frac{dx}{1+3x^2}$$

$u = \sqrt{3}x$

$du = \sqrt{3}\,dx$

$$\int \frac{dx}{1+(\sqrt{3}x)^2}$$

$$\frac{1}{\sqrt{3}}\int \frac{du}{1+u^2}$$

$$\frac{1}{\sqrt{3}}\tan^{-1}(u) + C$$

$$\boxed{\frac{1}{\sqrt{3}}\tan^{-1}(\sqrt{3}x)+C}$$

$$\int \frac{e^x}{\sqrt{1-e^{2x}}}dx$$

$u = e^x$

$du = \cancel{e^x}\,dx$

$$\int \frac{\cancel{e^x}\,dx}{\sqrt{1-(e^x)^2}}$$

$$\int \frac{du}{\sqrt{1-u^2}} = \boxed{\sin^{-1}(e^x)+C}$$

Problems with u-substitution:

$$\int_0^2 \frac{dx}{4+x^2}$$

$u = x/2$

$du = 1/2\, dx$

$$\frac{1}{4}\int_0^2 \frac{dx}{1+\frac{x^2}{4}}$$

$$\frac{1}{4}\int_0^2 \frac{dx}{1+\left(\frac{x}{2}\right)^2}$$

$$2 \cdot \frac{1}{4}\int_0^1 \frac{1}{1+u^2}\,du$$

$$\frac{1}{2}\tan^{-1}(u)\Big]_0^1$$

$$\frac{1}{2}\left(\tan^{-1}(1) - \tan^{-1}(0)\right)$$

$$\frac{1}{2}\left(\frac{\pi}{4}\right) = \pi/8$$

Problems with u-substitution:

$$\int \frac{1}{\sqrt{\underline{\underline{9}}-25x^2}}dx$$

$u = \frac{5}{3}x$

$du = \frac{5}{3}dx$

$$\frac{1}{3}\cdot\frac{3}{5}\int \frac{1}{\sqrt{1-u^2}}du$$

$$\int \frac{dx}{\sqrt{9\left(1-\frac{25}{9}x^2\right)}}$$

$$\boxed{\frac{1}{5}\sin^{-1}\left(\frac{5}{3}x\right)+C}$$

$$\frac{1}{3}\int \frac{dx}{\sqrt{1-\left(\frac{5}{3}x\right)^2}}$$

Problems with Completing the Square:

$$\int \frac{dx}{x^2+4x+17}$$

$$\int \frac{dx}{x^2+4x+4-4+17}$$

$$\frac{1}{13}\int \frac{dx}{\frac{(x+2)^2}{13}+\cancel{13}\,1}$$

$$\frac{1}{13}\int \frac{dx}{\left(\frac{x+2}{\sqrt{13}}\right)^2+1}$$

$$u=\frac{x+2}{\sqrt{13}}$$

$$du=\frac{1}{\sqrt{13}}\,dx$$

$$\frac{\sqrt{13}}{13}\int \frac{du}{u^2+1}$$

$$\boxed{\frac{\sqrt{13}}{13}\tan^{-1}\left(\frac{x+2}{\sqrt{13}}\right)+C}$$

Problems with Completing the Square:

$$\int \frac{dx}{\sqrt{-x^2+10x-24}}$$

$-x^2+10x-24$

$-(x^2-10x+24)$

$-(x^2-10x+25-25+24)$

$-((x-5)^2-1)$

$1-(x-5)^2$

$$\int \frac{dx}{\sqrt{1-(x-5)^2}}$$

$u=x-5$

$du=dx$

$$\int \frac{du}{\sqrt{1-u^2}}$$

$$\boxed{\sin^{-1}(x-5)+C}$$

Homework:

Inverse Trig Integrals WS – odds

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