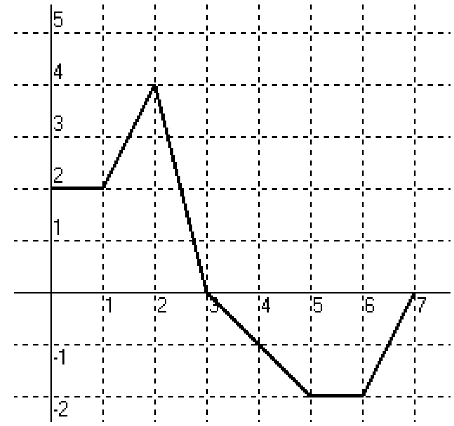
AP Calculus BC
Section 5.3 – FTC Free Response Questions



1. (Stewart – no calculator) Let $g(x)=\int_0^x f(t)dt$, where f is the function whose graph is shown to the right.

   a. Evaluate $g(0)$, $g(1)$, $g(2)$, $g(3)$, and $g(6)$.

   b. On what intervals is g increasing?

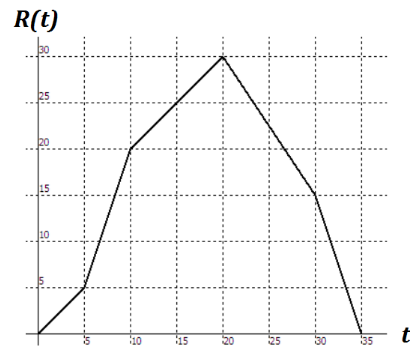
   c. Where does g have a maximum value?

   d. Evaluate $g'(2)$

   e. Find any points of inflection for g. Justify your answers.

2. (Lucia – calculator) Water is draining out of a tank at a variable rate as given by the chart and graph below.

t (min)	$R(t)$ (gallons/min)
0	0
5	5
10	20
20	30
30	15
35	0



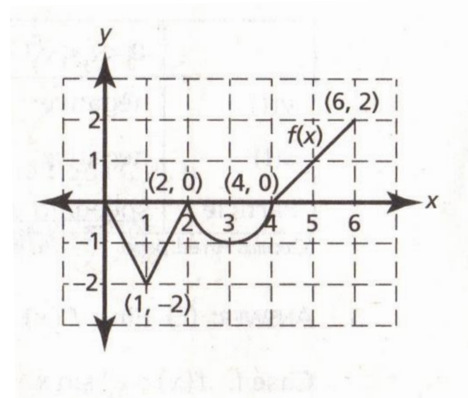
a. Approximate the volume of water that has leaked from the tank from 0 to 35 minutes using a Riemann sum with a right-hand end point for the five unequal intervals indicated by the chart.

b. Find the value of $\frac{1}{20}\int_{10}^{30} R(t)dt$. Using appropriate units, interpret the meaning of $\frac{1}{20}\int_{10}^{30} R(t)dt$ in the context of the problem.

c. Calculate $R'(25)$. Using appropriate units, interpret the meaning of $R'(25)$ in the context of the problem.

d. If the rate of the leak is modeled by $Q(t) = 16.78\sin(0.15t - 1.25) + 14.6$, at what time is the water leaking the fastest on the interval [0, 35]?

3. (Lucia – no calculator) Let f by a function defined in the closed interval $0 \le x \le 6$. The graph of f consists of three line segments and a semicircle. Let $g(x) = 3 + \int_2^x f(t)dt$.



   a. Find $g(1), g'(1)$, and $g''(1)$.

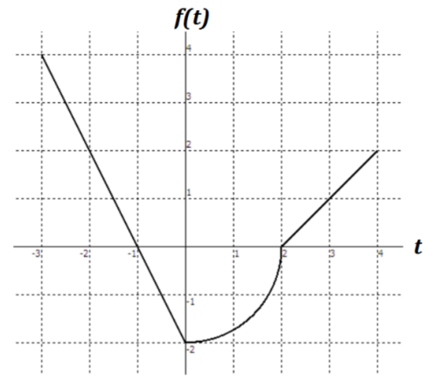
   b. What is the average rate of change of $g(x)$ in the interval $2 \le x \le 6$?

   c. What is the average value of $g'(x)$ on the same interval as part b)?

   d. Identify the x – coordinate(s) of any relative extrema for g. Justify your answers.

   e. Identify the x – coordinate(s) of any points of inflection for g. Justify your answers.

4. (Lucia – no calculator) The graph of $f(t)$, a continuous function defined on the interval $-3 \le t \le 4$, consists of two line segments and a quarter circle, as show in the figure. Let $g(x) = \int_{-3}^{x} f(t)dt$.



   a. Evaluate $g(0)$ and $g(4)$.

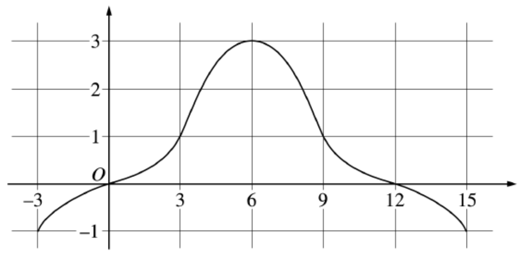
   b. Find the x – coordinate of the absolute maximum and absolute minimum of $g(x)$. Justify your answers.

   c. Does $\lim_{x \to 2} g''(x)$ exist? Give a reason for your answer.

   d. Find the x – coordinates of all inflection points of $g(x)$. Justify your answer.

AP Calculus BC
Section 5.3 – FTC Free Response Questions

5. (2002B BC) The graph of a differentiable function *f* on the closed interval $[-3,15]$ is shown in the figure. The graph of *f* has a horizontal tangent at $x = 6$. Let $g(x)=5+\int_6^x f(t)dt$ for $-3\le x\le 15$.



Graph of f

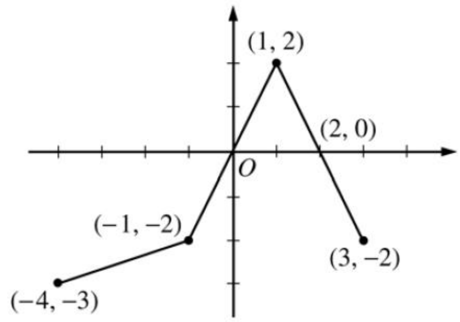
a. Find $g(6)$, $g'(6)$, and $g''(6)$.

b. On what intervals is g decreasing? Justify your answer.

c. On what intervals is the graph of g concave down? Justify your answer.

d. Find a trapezoidal approximation of $\int_{-3}^{15} f(t)dt$ using six subintervals of equal size.

6. (2005B BC4) The graph of the function f consists of three line segments.



Graph of f

a. Let g be the function given by $g(x)=\int_{-4}^{x} f(t)dt$. For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

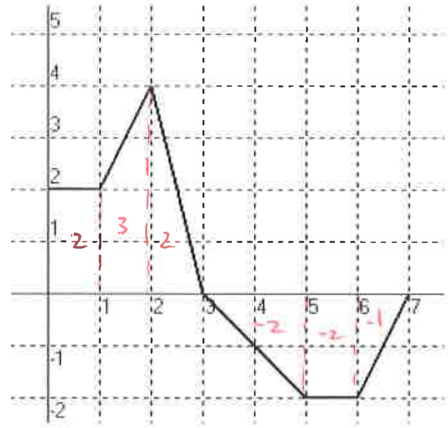
b. For the function g defined in part (a), find the x – coordinate of each point of inflection of the graph of g on the open interval $-4<x<3$. Explain your reasoning.

c. Let h be the function given by $h(x)=\int_{x}^{3} f(t)dt$. Find all values of x in the closed interval $-4\le x\le 3$ for which $h(x)=0$.

d. For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

AP Calculus BC
Section 5.3 – FTC Free Response Questions

1. (Stewart – no calculator) Let $g(x)=\int_0^x f(t)dt$, where f is the function whose graph is shown to the right.



a. Evaluate $g(0)$, $g(1)$, $g(2)$, $g(3)$, and $g(6)$.

$g(0)=\int_0^0 f(t)dt=0$ $\quad$ $g(3)=\int_0^3 f(t)dt=7$

$g(1)=\int_0^1 f(t)dt=2$ $\quad$ $g(6)=\int_0^6 f(t)dt=3$

$g(2)=\int_0^2 f(t)dt=5$

b. On what intervals is g increasing?

g IS INCR on $[0,3]$ SINCE $g'=f>0$ on $(0,3)$.

c. Where does g have a maximum value?

$g'=f=0$ AT $x=3$

USE CANDIDATES TEST:

$g(0)=0$

$g(3)=7$

$g(7)=2$

MAX VALUE IS AT $X=3$.

d. Evaluate $g'(2)$

$g'(2)=f(2)=4$

e. Find any points of inflection for g. Justify your answers.

$g''=f'$ CHANGES SIGN AT $X=2 \Rightarrow$ P.O.I. @ $X=2$.

AP Calculus BC
Section 5.3 – FTC Free Response Questions

2. (Lucia – calculator) Water is draining out of a tank at a variable rate as given by the chart and graph below.

t (min)	$R(t)$ (gallons/min)
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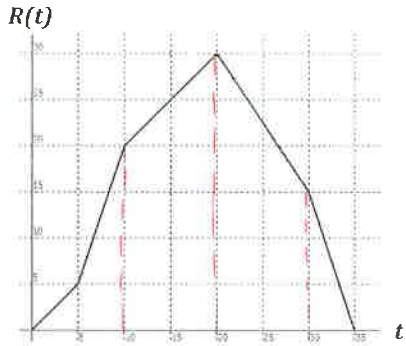


a. Approximate the volume of water that has leaked from the tank from 0 to 35 minutes using a Riemann sum with a right-hand end point for the five unequal intervals indicated by the chart.

$5(5)+5(20)+10(30)+10(15)+5(0) = 575$ GALLONS.

b. Find the value of $\frac{1}{20}\int_{10}^{30} R(t)dt$. Using appropriate units, interpret the meaning of $\frac{1}{20}\int_{10}^{30} R(t)dt$ in the context of the problem.

$\frac{1}{20}\int_{10}^{30} R(t)\,dt = \frac{1}{20}(600-50-75) = 23.75 \frac{\text{GAL}}{\text{MIN}}$

THE AVG. RATE THAT H_2O LEAKS IS INCREASING 23.75 GAL/MIN FROM $t=10$ TO $t=30$ MIN.

c. Calculate $R'(25)$. Using appropriate units, interpret the meaning of $R'(25)$ in the context of the problem.

$R'(25) = \frac{15-30}{10} = -1.5 \frac{\text{GAL}}{\text{MIN}^2}$

THE RATE H_2O IS LEAVING THE TANK IS DECREASING 1.5 $\frac{\text{GAL}}{\text{MIN}^2}$ AT $t=25$ MIN.

d. If the rate of the leak is modeled by $Q(t) = 16.78\sin(0.15t - 1.25) + 14.6$, at what time is the water leaking the fastest on the interval [0, 35]?

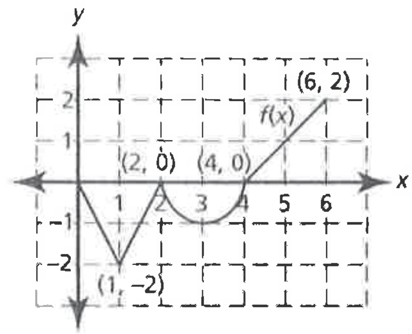
$Q'(t) = 0$ AT $t = 18.805$

$Q(0) = -15.9239$
$Q(18.805) = 16.7799$
$Q(35) = -12.699$

H_2O IS LEAKING FASTEST AT $t = 18.805$.

AP Calculus BC
Section 5.3 – FTC Free Response Questions

3. (Lucia – no calculator) Let f by a function defined in the closed interval $0 \le x \le 6$. The graph of f consists of three line segments and a semicircle. Let $g(x) = 3 + \int_2^x f(t)\,dt$.



a. Find $g(1)$, $g'(1)$, and $g''(1)$. $g' = f$

$g(1) = 3 + \int_2^1 f(t)\,dt = 3 + 1 = 4$

$g'(1) = f(1) = -2$

$g''(1) = f'(1) \rightarrow$ DNE.

THESE ARE THE SAME

b. What is the average rate of change of $g(x)$ in the interval $2 \le x \le 6$?

$\dfrac{g(6) - g(2)}{6-2} = \dfrac{2 - \pi/2}{4} = \dfrac{4-\pi}{8} = \dfrac{1}{2} - \dfrac{\pi}{8}$

c. What is the average value of $g'(x)$ on the same interval as part b)?

$g'_{AVE} = \frac{1}{4}\int_2^6 g'(t)\,dt = \frac{1}{4}\int_2^6 f(t)\,dt$

$= \frac{1}{4}\left(g(6) - g(2)\right)$

$= \dfrac{4-\pi}{8}$

d. Identify the x – coordinate(s) of any relative extrema for g. Justify your answers.

$g' = f = 0$ AT $x = 2, 4$

g': − − on (0, 2), − − on (2, 4), + + on (4, 6); marks at 0, 2, 4, 6

g' CHANGES FROM − TO + AT $x = 4$

$\Rightarrow g$ HAS REL. MIN AT $x = 4$.

e. Identify the x – coordinate(s) of any points of inflection for g. Justify your answers.

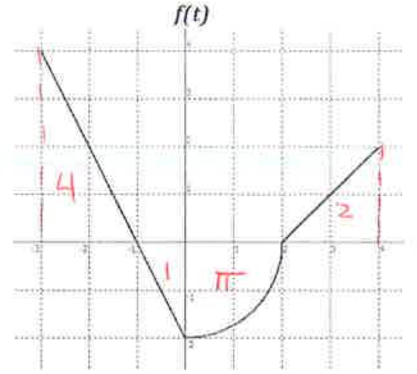
g'' CHANGES SIGN AT $x = 1, 2, 3 \Rightarrow g$ HAS P.O.I. AT $x = 1, 2, 3$

$g'' = f'$

g'': − on (0, 1), + on (1, 2), − on (2, 3), + on (3, 4), + on (4, 6); marks at 0, 1, 2, 3, 4, 6

AP Calculus BC
Section 5.3 – FTC Free Response Questions

4. (Lucia – no calculator) The graph of $f(t)$, a continuous function defined on the interval $-3 \le t \le 4$, consists of two line segments and a quarter circle, as show in the figure. Let $g(x) = \int_{-3}^{x} f(t)dt$.



a. Evaluate $g(0)$ and $g(4)$.

$g(0) = \int_{-3}^{0} f(t)dt = 3$

$g(4) = \int_{-3}^{4} f(t)dt = 5 - \pi$

b. Find the x – coordinate of the absolute maximum and absolute minimum of $g(x)$. Justify your answers.

$g' = f = 0$ AT $x = -1, 2$.

$g(-1) = \int_{-3}^{-1} f(t)dt = 4$

$g(2) = \int_{-3}^{2} f(t)dt = 3 - \pi$

$g(4) = 5 - \pi$

$g(-3) = 0$

ABS. MAX IS AT $x = -1$

ABS. MIN IS AT $x = 2$.

c. Does $\lim_{x \to 2} g''(x)$ exist? Give a reason for your answer.

$g'' = f'$

$\lim_{x \to 2^+} f' = 1$

$\lim_{x \to 2^-} f' \Rightarrow \infty$

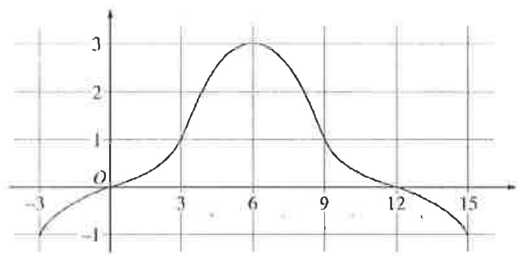
SINCE $\lim_{x \to 2^+} f' \neq \lim_{x \to 2^-} f' \Rightarrow \lim_{x \to 2} g''$ DNE.

d. Find the x – coordinates of all inflection points of $g(x)$. Justify your answer.

$g'' = f'$ CHANGES SIGN AT $x = 0 \Rightarrow$ P.O.I. @ $x = 0$.

5. (2002B BC) The graph of a differentiable function f on the closed interval $[-3,15]$ is shown in the figure. The graph of f has a horizontal tangent at $x = 6$. Let $g(x)=5+\int_6^x f(t)dt$ for $-3\le x\le 15$.

☆ NON CALCULATOR



Graph of f

a. Find $g(6)$, $g'(6)$, and $g''(6)$.

$g(6) = 5 + \int_6^6 f(t)dt = 5$

$g'(6) = f(6) = 3$

$g''(6) = f'(6) = 0$

b. On what intervals is g decreasing? Justify your answer.

$g' = f < 0$ on $(-3,0) \cup (12,15) \Rightarrow g$ IS DECREASING on $[-3,0] \cup [12,15]$

c. On what intervals is the graph of g concave down? Justify your answer.

$g'' = f' < 0$ on $(6,15) \Rightarrow g$ IS CONCAVE DOWN on $(6,15)$

d. Find a trapezoidal approximation of $\int_{-3}^{15} f(t)dt$ using six subintervals of equal size.

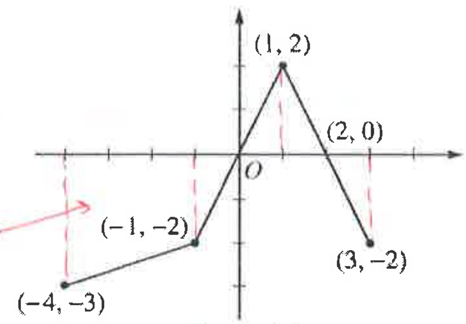
$\frac{1}{2}(3)\left[-1 + \cancel{2(0)} + 2(1) + 2(3) + 2(1) + \cancel{2(0)} + -1\right]$

$= 12$

AP Calculus BC
Section 5.3 – FTC Free Response Questions

6. (2005B BC4) The graph of the function f consists of three line segments.

★NON CALCULATOR★



Graph of f

$\frac{1}{2}(3)(-3+-2)$

$-15/2$

a. Let g be the function given by $g(x)=\int_{-4}^{x} f(t)dt$. For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

$g(-1)=\int_{-4}^{-1} f(t)dt = -15/2$ $\quad g'(-1)=f(-1)=-2$ $\quad g''(-1)=f'(-1)\rightarrow$ DNE

b. For the function g defined in part (a), find the x – coordinate of each point of inflection of the graph of g on the open interval $-4<x<3$. Explain your reasoning.

$g''=f'$ CHANGES SIGN AT $x=1$ $\Rightarrow$ P.O.I. AT $x=1$

c. Let h be the function given by $h(x)=\int_{x}^{3} f(t)dt$. Find all values of x in the closed interval $-4\le x\le 3$ for which $h(x)=0$.

$h(x)=0$ AT $x=3 \rightarrow \int_3^3 f(t)dt=0$

$h(x)=0$ AT $x=1 \rightarrow \int_1^3 f(t)dt=0$

$h(x)=0$ AT $x=-1 \rightarrow \int_{-1}^3 f(t)dt=0$

d. For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

h DECR IF $h'=-f<0$

$\Rightarrow f>0$ on $(0,2)$

$\Rightarrow h$ DECR on $[0,2]$