AP Calculus BC
Chapter 5 – AP Exam Problems

Riemann Sums – Rectangle Approximation Method

t (sec)	0	2	4	6
$a(t)$ ft/sec^2	5	2	8	3

1. The data for acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table above. If the velocity at $t = 0$ is 11 feet per second, the approximate value of the velocity at $t = 6$, computed using a left-hand Riemann sum with three subintervals of equal length, is

   A) 26 ft/sec
   B) 30 ft/sec
   C) 37 ft/sec
   D) 39 ft/sec
   E) 41 ft/sec

2. If n is a positive integer, then $\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \cdots + \left(\frac{3n}{n}\right)^2\right]$ can be expressed as

   A) $\int_0^1 \frac{1}{x^2}\,dx$
   B) $\int_0^1 3\left(\frac{1}{x}\right)^2 dx$
   C) $\int_0^3 \left(\frac{1}{x}\right)^2 dx$
   D) $\int_0^3 x^2\,dx$
   E) $3\int_0^3 x^2\,dx$

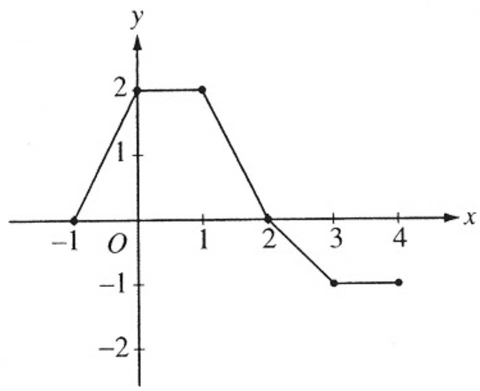
3. Find $\lim_{n\to\infty} \frac{1}{n}\left[\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \cdots + \sqrt{\frac{n}{n}}\right] =$

   A) $\frac{1}{2}\int_0^1 \frac{1}{\sqrt{x}}\,dx$
   B) $\int_0^1 \sqrt{x}\,dx$
   C) $\int_0^1 x\,dx$
   D) $\int_1^2 x\,dx$
   E) $2\int_1^2 x\sqrt{x}\,dx$

Geometric Solutions to Definite Integrals

4. Let f be a continuous function on the closed interval [0, 2]. If $2 \le f(x) \le 4$, then the greatest possible value of $\int_0^2 f(x)\,dx$ is

   A) 0
   B) 2
   C) 4
   D) 8
   E) 16



5. The graph of a piecewise-linear function *f*, for $-1 \le x \le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\,dx$?

   A) 1 B) 2.5 C) 4 D) 5.5 E) 8

6. Find $\int_{0}^{2} \sqrt{4-x^2}\,dx =$

   A) $\frac{8}{3}$ B) $\frac{16}{3}$ C) π D) 2π E) 4π

Properties of Definite Integrals

7. If $\int_{-1}^{1} e^{-x^2}dx = k$, then $\int_{-1}^{0} e^{-x^2}dx =$

   A) $-2k$ B) $-k$ C) $-\frac{k}{2}$ D) $\frac{k}{2}$ E) $2k$

8. Let *f* and *g* have continuous first and second derivatives everywhere. If $f(x) \le g(x)$ for all real *x*, which of the following must be true?

   I. $f'(x) \le g'(x)$ for all real *x*
   II. $f''(x) \le g''(x)$ for all real *x*
   III. $\int_{0}^{1} f(x)\,dx \le \int_{0}^{1} g(x)\,dx$

   A) None
   B) I only
   C) III only
   D) I and II only
   E) I, II, and III

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9. If $\int_1^{10} f(x)\,dx = 4$ and $\int_{10}^{3} f(x)\,dx = 7$, then $\int_1^{3} f(x)\,dx =$

A) -3 B) 0 C) 3 D) 10 E) 11

10. If *f* and *g* are continuous functions, and if $f(x) \ge 0$ for all real numbers *x*, which of the following must be true?

I. $\int_a^b f(x)g(x)\,dx = \left(\int_a^b f(x)\,dx\right)\left(\int_a^b g(x)\,dx\right)$

II. $\int_a^b \left(f(x)+g(x)\right)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

III. $\int_a^b \sqrt{f(x)}\,dx = \sqrt{\int_a^b f(x)\,dx}$

A) I only
B) II only
C) III only
D) II and III only
E) I, II, and III

11. If *f* is a function such that $f'(x)$ exists for all *x* and $f(x) > 0$ for all *x*, which of the following is **NOT** necessarily true?

A) $\int_{-1}^{1} f(x)\,dx > 0$

B) $\int_{-1}^{1} 2f(x)\,dx = 2\int_{-1}^{1} f(x)\,dx$

C) $\int_{-1}^{1} f(x)\,dx = 2\int_{0}^{1} f(x)\,dx$

D) $\int_{-1}^{1} f(x)\,dx = -\int_{1}^{-1} f(x)\,dx$

E) $\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$

12. If $\int_1^4 f(x)\,dx = 6$, what is the value of $\int_1^4 f(5-x)\,dx$?

A) 6 B) 3 C) 0 D) -1 E) -6

13. If $\int_a^b f(x)\,dx = 5$ and $\int_a^b g(x)\,dx = -1$, which of the following must be true?

I. $f(x) > g(x)$ for $a \le x \le b$

II. $\int_a^b \left[f(x)+g(x)\right]\,dx = 4$

III. $\int_a^b \left[f(x)g(x)\right]\,dx = -5$

A) I only
B) II only
C) III only
D) II and III only
E) I, II, and III

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14. If $f(x)=g(x)+7$ for $3\le x\le 5$, then $\int_3^5 \left[f(x)+g(x)\right]dx =$

A) $2\int_3^5 g(x)\,dx+7$

B) $2\int_3^5 g(x)\,dx+14$

C) $2\int_3^5 g(x)\,dx+28$

D) $\int_3^5 g(x)\,dx+7$

E) $\int_3^5 g(x)\,dx+14$

The Trapezoidal Method for Area Approximations

15. If the definite integral $\int_0^2 e^{x^2}\,dx$ is first approximated by using two <u>inscribed</u> rectangles of equal width and then approximated by using the trapezoidal rule with $n = 2$, the difference between the two approximations is

A) 53.60 B) 30.51 C) 27.80 D) 26.80 E) 12.78

x	2	5	7	8
$f(x)$	10	30	40	20

16. The function f is continuous on the closed interval [2, 8] and has values that are given in the table above. Using the subintervals [2, 5], [5, 7], and [7, 8], what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

A) 110 B) 130 C) 160 D) 190 E) 210

17. If three equal subdivisions of $[-4, 2]$ are used, what is the trapezoidal approximation of $\int_{-4}^{2} \frac{e^{-x}}{2}\,dx$?

A) $e^2+e^0+e^{-2}$

B) $e^4+e^2+e^0$

C) $e^4+2e^2+2e^0+e^{-2}$

D) $\frac{1}{2}\left(e^4+e^2+e^0+e^{-2}\right)$

E) $\frac{1}{2}\left(e^4+2e^2+2e^0+e^{-2}\right)$

Simpson's Method for Area Approximations

18. Let R be the region in the first quadrant enclosed by the x-axis and the graph of $y=\ln\left(1+2x-x^2\right)$. If Simpson's Rule with 2 subintervals is used to approximate the area R, the approximation is

A) 0.462 B) 0.693 C) 0.924 D) 0.986 E) 1.850

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The Fundamental Theorem of Calculus

19. Find $\frac{d}{dx}\int_{2}^{x}\sqrt{1+t^2}\,dt =$

A) $\frac{x}{\sqrt{1+x^2}}$ B) $\sqrt{1+x^2}-5$ C) $\sqrt{1+x^2}$

D) $\frac{x}{\sqrt{1+x^2}}-\frac{1}{\sqrt{5}}$ E) $\frac{1}{2\sqrt{1+x^2}}-\frac{1}{2\sqrt{5}}$

20. For all $x > 1$, if $f(x)=\int_{1}^{x}\frac{1}{t}\,dt$, then $f'(x)=$

A) 1 B) $\frac{1}{x}$ C) $\ln x-1$ D) $\ln x$ E) e^x

21. Find $\frac{d}{dx}\int_{0}^{x}\cos(2\pi u)\,du$

A) 0 B) $\frac{1}{2\pi}\sin x$ C) $\frac{1}{2\pi}\cos(2\pi x)$ D) $\cos(2\pi x)$ E) $2\pi\cos(2\pi x)$

22. If $F(x)=\int_{0}^{x}\sqrt{t^3+1}\,dt$, then $F'(2)=$

A) -3 B) -2 C) 2 D) 3 E) 18

23. Let $f(x)=\int_{-2}^{x^2-3x}e^{t^2}\,dt$. At what value of x is $f(x)$ a minimum?

A) No values B) $\frac{1}{2}$ C) $\frac{3}{2}$ D) 2 E) 3

24. If $F(x)=\int_{1}^{x^2}\sqrt{1+t^3}\,dt$, then $F'(x)=$

A) $2x\sqrt{1+x^6}$ C) $\sqrt{1+x^6}$ E) $\int_{1}^{x^2}\frac{3t^2}{2\sqrt{1+t^3}}\,dt$

B) $2x\sqrt{1+x^3}$ D) $\sqrt{1+x^3}$

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Average Value of a Function

25. The average value of $f(x)=x^2\sqrt{x^3+1}$ on the closed interval [0, 2] is

A) $\frac{26}{9}$ B) $\frac{13}{3}$ C) $\frac{26}{3}$ D) 13 E) 26

26. What is the average value of *y* for the part of the curve $y=3x-x^2$ which is in the <u>first quadrant</u>?

A) -6 B) -2 C) $\frac{3}{2}$ D) $\frac{9}{4}$ E) $\frac{9}{2}$

27. The average value of $\frac{1}{x}$ on the closed interval [1, 3] is

A) $\frac{9}{2}$ B) $\frac{2}{3}$ C) $\frac{\ln 2}{2}$ D) $\frac{\ln 3}{2}$ E) $\ln 3$

28. If $\frac{dy}{dx}=\frac{1}{x}$, then the average rate of change of *y* with respect to *x* on the closed interval [1, 4] is

A) $-\frac{1}{4}$ B) $\frac{1}{2}\ln 2$ C) $\frac{2}{3}\ln 2$ D) $\frac{2}{5}$ E) 2

29. Let $f(x)=14\pi x^2$ and $g(x)=k^2\sin\left(\frac{\pi x}{2k}\right)$ for $k>0$.

a) Find the average value of *f* on [1, 4].
b) For what value of *k* will the average value of *g* on [0, *k*] be equal to the average value of *f* on [1, 4]?

30. Let *f* be a function such that $f''(x)=6x+8$.

a) Find $f(x)$ if the graph of *f* is tangent to the line $3x-y=2$ at the point $(0,-2)$.
b) Find the average value of $f(x)$ on the closed interval $[-1,1]$.

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31. Let f be the function that is defined for all real numbers x and that has the following properties.

(i) $f''(x) = 24x - 18$ (ii) $f'(1) = -6$ (iii) $f(2) = 0$

a) Find each x such that the line tangent to the graph of f at $(x, f(x))$ is horizontal.
b) Write an expression for $f(x)$.
c) Find the average value of f on the interval $-1 \le x \le 3$.

32. Let f be the function given by $f(x) = x^3 - 6x^2 + p$, where p is an arbitrary constant.

a) Write an expression for $f'(x)$ and use it to find the relative maximum and minimum values of f in terms of p. Show the analysis that leads to your conclusion.
b) For what values of the constant p does f have 3 distinct real roots?
c) Find the value of p such that the average value of f over the closed interval $[-1, 2]$ is 1.

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t	$R(t)$
hours	gallons/hour
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

33. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate measured every 3 hours for a 24-hour period.

a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)dt$.
b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.
c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}\left(768 + 23t - t^2\right)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

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Mean Value Theorem for Integrals

34. If f is continuous on the closed interval $[a, b]$, then there exists c such that $a < c < b$ and $\int_a^b f(x)dx =$

A) $\dfrac{f(c)}{b-a}$ C) $f(b)-f(a)$ E) $f(c)(b-a)$

B) $\dfrac{f(b)-f(a)}{b-a}$ D) $f'(c)(b-a)$

Answer Key

1.	E	1998	BC	#91	26%				
2.	D	1985	BC	#45	34%				
3.	B	1988	BC	#41	35%				
4.	D	1985	AB	#40	41%				
5.	B	1998	AB	#2	43%				
6.	C	1988	BC	#31	34%				
7.	D	1985	AB	#9	62%				
8.	C	1985	AB	#38	25%				
9.	E	1988	AB	#39	34%				
10.	B	1993	AB	#12	68%				
11.	C	1988	BC	#26	68%				
12.	A	1988	BC	#42	15%				
13.	B	1993	BC	#32	57%				
14.	B	1998	BC	#82	48%				
15.	D	1993	AB	#36	10%				
16.	C	1998	AB	#85	46%				
17.	E	1988	BC	#18	43%				
18.	C	1993	BC	#40	22%				
19.	C	1985	AB	#42	29%				
20.	B	1988	AB	#25	64%				
21.	D	1993	AB	#41	54%				
22.	D	1998	AB	#15	71%				
23.	C	1993	BC	#41	32%				
24.	A	1988	BC	#14	44%				
25.	A	1985	AB	#44	38%				
26.	C	1988	AB	#36	60%				
27.	D	1988	BC	#21	83%				
28.	C	1993	BC	#39	52%				
29.		1985	BC	#3	FRQ				
30.		1989	BC	#1	FRQ				
31.		1991	AB	#1	FRQ				
32.		1997	AB	#4	FRQ				
33.		1999	BC	#3	FRQ				
34.	E	1993	BC	#44	47%				