Area Approximation Methods

Chapter 5

Suppose you are driving to Pitt for a college visit. You record your speed (miles per hour) at various times (hours.) Assume that $v \geq 0$ for $t \geq 0$.

t	$v(t)$
0	10
1	58
2	67
3	95
4	0
5	55
6	30

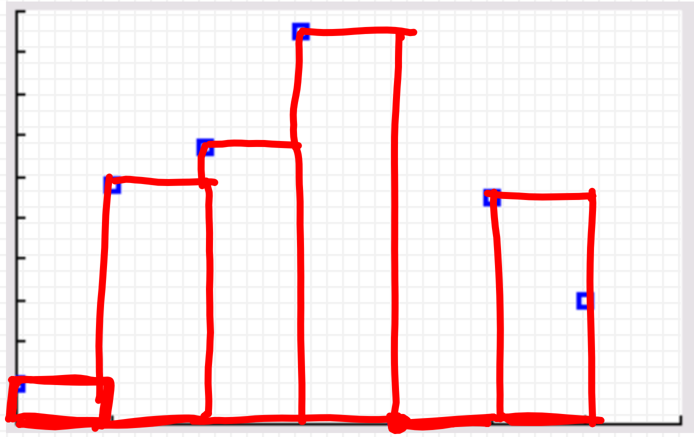
Approximately how far away is Pitt?

DISTANCE TRAVELED IS AREA UNDER $v(t)$.

$$D = \int_0^6 v(t)\,dt$$

Left Rectangular Approximation Method (LRAM)

t	$v(t)$
0	10
1	58
2	67
3	95
4	0
5	55
6	~~30~~



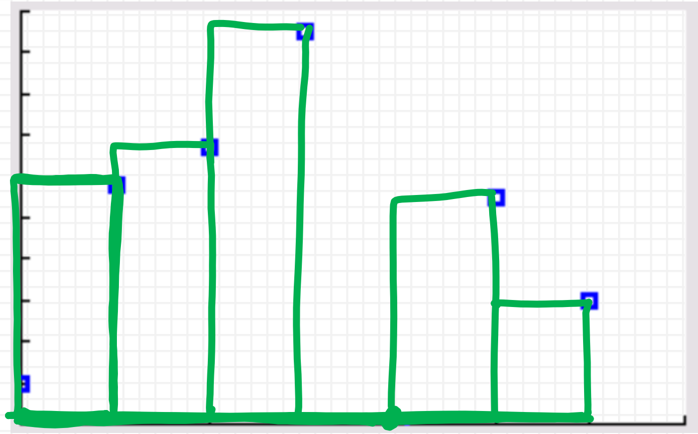
$$\int_0^6 v(t)\,dt \approx 1(10)+1(58)+1(67)+1(95)+1(0)+1(55)$$

$$= 1(10+58+67+95+0+55)$$

$$= 285 \text{ miles}$$

Right Rectangular Approximation Method (RRAM)

t	$v(t)$
0	~~10~~
1	58
2	67
3	95
4	0
5	55
6	30



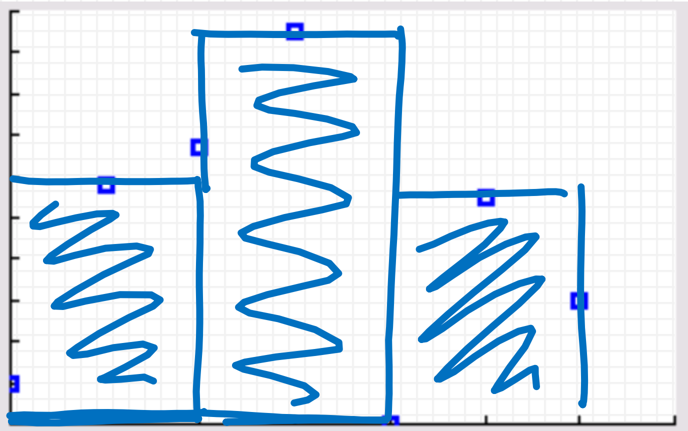
$$\int_0^6 v(t)\,dt \approx$$

$$1(58+67+95+0+55+30)$$

$$= 305 \text{ miles.}$$

Midpoint Rectangular Approximation Method (MRAM)

t	$v(t)$
0	10
1	58
2	67
3	95
4	0
5	55
6	30



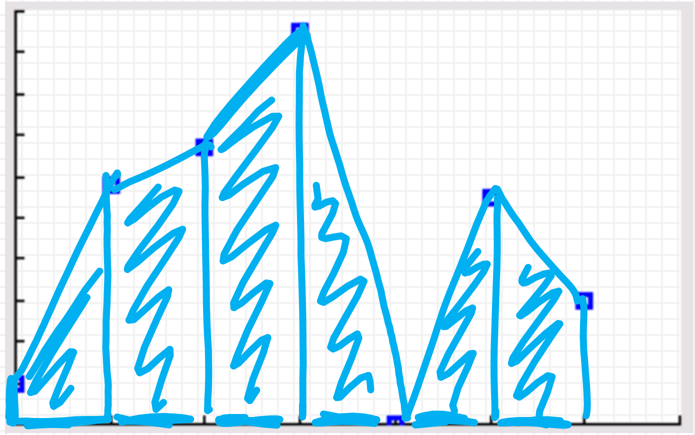
$$\int_0^6 v(t) \approx$$

$$2(58+95+55)$$

$$= 416 \text{ miles.}$$

Trapezoidal Approximation Method (TRAP)

t	$v(t)$
0	10
1	58
2	67
3	95
4	0
5	55
6	30

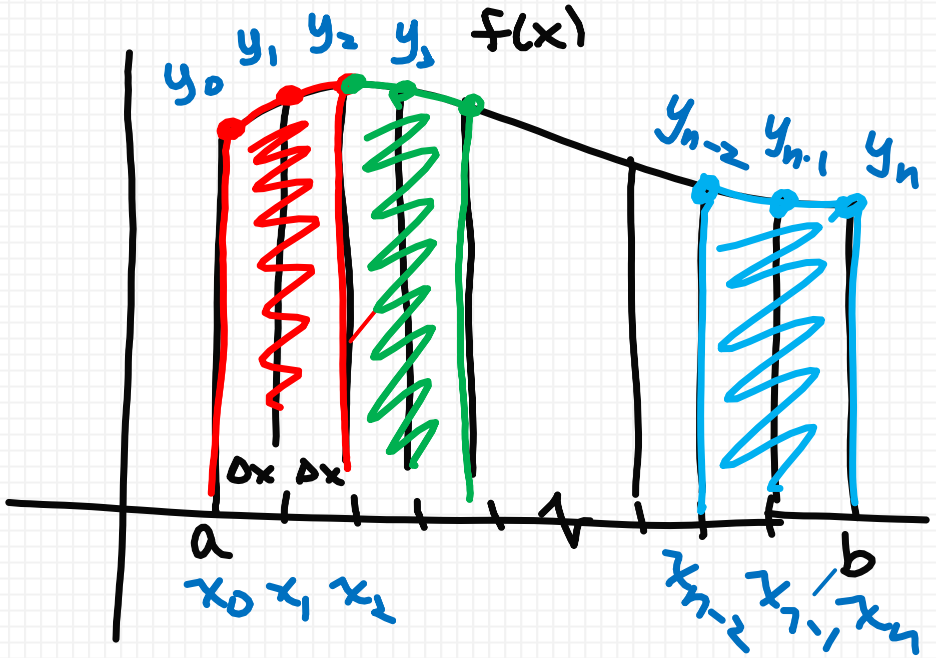


$$\int_0^6 v(t)\,dt$$

$$\approx \tfrac{1}{2}(1)(10+58) + \tfrac{1}{2}(1)(58+67) + \tfrac{1}{2}(1)(67+95) + \tfrac{1}{2}(1)(95+0) + \tfrac{1}{2}(1)(0+55) + \tfrac{1}{2}(1)(55+30)$$

$$= \tfrac{1}{2}(1)\left[10 + 2(58) + 2(67) + 2(95) + 2(0) + 2(55) + 30\right] = 295 \text{ miles.}$$

Simpson's Rule (SIMP) : EVEN NUMBER (n) OF EQUAL SUBINTERVALS



$$\Delta x = \frac{b-a}{n}$$

$$\frac{1}{3}\Delta x(y_0 + 4y_1 + y_2) + \frac{1}{3}\Delta x(y_2 + 4y_3 + y_4)$$

$$+ \cdots\cdots + \frac{1}{3}\Delta x(y_{n-2} + 4y_{n-1} + y_n)$$

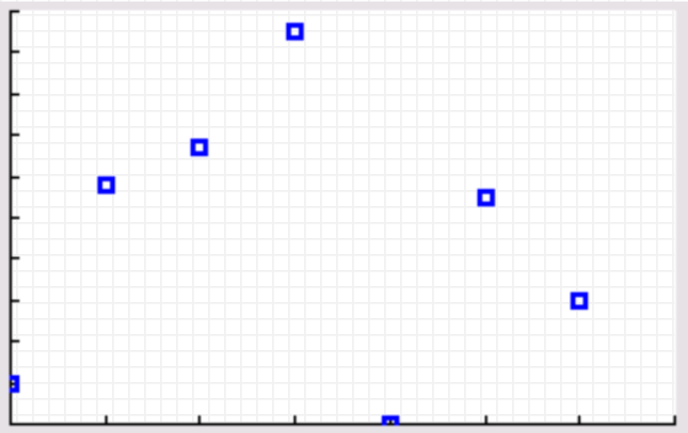
$$= \frac{1}{3}\Delta x\Big(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots$$

$$\cdots + 2y_{n-2} + 4y_{n-1} + y_n\Big)$$

ok! → **Simpson's Rule (SIMP)**

t	v(t)
0	10
1	58
2	67
3	95
4	0
5	55
6	30

$n = 6 \qquad \Delta t = 1$



$$\int_0^6 v(t)\,dt$$

$$\approx \frac{1}{3}(1)\left[10 + 4(58) + 2(67) + 4(95) + 2(0) + 4(55) + 30\right]$$

$$= 335\tfrac{1}{3} \text{ miles.}$$

What if some of the data is missing?

t	$v(t)$
0	10
2	67
3	95
6	30

$LRAM = 2(10) + 1(67) + 3(95)$

$RRAM = 2(67) + 1(95) + 3(30)$

$TRAP = \frac{1}{2}(2)(10+67) + \frac{1}{2}(1)(67+95) + \frac{1}{2}(3)(95+30)$

~~MRAM~~

~~Simp~~

Approximate with all methods using 4 subintervals of equal length. $\Delta x = 1$

$$\int_0^4 e^{-x^2}\,dx$$

$$LRAM = 1\left(e^{0} + e^{-1} + e^{-4} + e^{-9}\right)$$

$$RRAM = 1\left(e^{-1} + e^{-4} + e^{-9} + e^{-16}\right)$$

$$MRAM = 1\left(e^{-(1/2)^2} + e^{-(3/2)^2} + e^{-(5/2)^2} + e^{-(7/2)^2}\right)$$

$$TRAP = \frac{1}{2}(1)\left[e^{0} + 2e^{-1} + 2e^{-4} + 2e^{-9} + e^{-16}\right]$$

$$SIMP = \frac{1}{3}(1)\left[e^{0} + 4e^{-1} + 2e^{-4} + 4e^{-9} + e^{-16}\right]$$

Homework:

Section 5.1 – Area Approximation Methods WS