Calculus I
Section 5.8 – Definite Integrals Using Substitution

Evaluate the following definite integrals using the First Fundamental Theorem.

1. $\int_0^3 \sqrt{y+1}\,dy$

2. $\int_0^1 r\sqrt{1-r^2}\,dr$

3. $\int_0^{\pi/4} \tan x \sec^2 x\,dx$

4. $\int_0^{\pi} 3\cos^2 x \sin x\,dx$

5. $\int_0^1 t^3(1+t^4)^3\,dt$

6. $\int_0^{\sqrt{7}} t(t^2+1)^{1/3}\,dt$

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7. $\int_{-1}^{1} \frac{5r}{(4+r^2)^2} dr$

8. $\int_{0}^{1} \frac{10\sqrt{v}}{(1+v^{3/2})^2} dv$

9. $\int_{0}^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}} dx$

10. $\int_{0}^{2\pi} \frac{\cos z}{\sqrt{4+3\sin z}} dz$

11. $\int_{0}^{\pi/6} \cos^{-3} 2x \sin 2x dx$

12. $\int_{0}^{\pi/6} (1-\cos 3t)\sin 3t dt$

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Evaluate the following definite integrals using the First Fundamental Theorem.

1. $\int_0^3 \sqrt{y+1}\,dy = \boxed{\frac{14}{3}}$

$u=y+1$
$du=dy$
$\int_1^4 u^{1/2}\,du$
$\frac{2}{3}u^{3/2}\Big]_1^4 = \frac{2}{3}\left[4^{3/2}-1^{3/2}\right]$

2. $\int_0^1 r\sqrt{1-r^2}\,dr = \boxed{\frac{1}{3}}$

$u=1-r^2$
$du=-2r\,dr$
$-\frac{1}{2}\int_1^0 u^{1/2}\,du$
$+\frac{1}{3}u^{3/2}\Big]_0^1 = \frac{1}{3}(1-0)$

3. $\int_0^{\pi/4} \tan x \sec^2 x\,dx = \boxed{\frac{1}{2}}$

$u=\tan x$
$du=\sec^2 x\,dx$
$\int_0^1 u\,du$
$\frac{1}{2}u^2\Big]_0^1 = \frac{1}{2}(1-0)$

4. $\int_0^{\pi} 3\cos^2 x \sin x\,dx = \boxed{2}$

$u=\cos x$
$-du=+\sin x\,dx$
$+\int_{-1}^{+1} u^2\,du$
$3\cdot\frac{1}{3}u^3\Big]_{-1}^{1} = \frac{3}{3}(1+1)$

5. $\int_0^1 t^3(1+t^4)^3\,dt = \boxed{15/16}$

$u=1+t^4$
$du=4t^3$
$\frac{1}{4}\int_1^2 u^3\,du$
$\frac{1}{16}u^4\Big]_1^2 = \frac{1}{16}(16-1)$

6. $\int_0^{\sqrt{7}} t(t^2+1)^{1/3}\,dt = \boxed{45/8}$

$u=t^2+1$
$du=2t\,dt$
$\frac{1}{2}\int_1^8 u^{1/3}\,du$
$\frac{3}{8}u^{4/3}\Big]_1^8 = \frac{3}{8}(8^{4/3}-1)$

7. $\int_{-1}^1 \frac{5r}{(4+r^2)^2}\,dr = \boxed{0}$

$u=4+r^2$
$du=2r\,dr$
$\frac{5}{2}\int_5^5 u^{-2}\,du$

8. $\int_0^1 \frac{10\sqrt{v}}{(1+v^{3/2})^2}\,dv = \boxed{\frac{10}{3}}$

$u=1+v^{3/2}$
$du=\frac{3}{2}v^{1/2}\,dv$
$\frac{20}{3}\int_1^2 u^{-2}\,du$
$-\frac{20}{3}u^{-1}\Big]_1^2 = -\frac{20}{3}\left(\frac{1}{2}-1\right)$

9. $\int_0^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}}\,dx = \boxed{4}$

$u=x^2+1$
$du=2x\,dx$
$2\int_1^4 u^{-1/2}\,du$
$4u^{1/2}\Big]_1^4 = 4(2-1)$

10. $\int_0^{2\pi} \frac{\cos z}{\sqrt{4+3\sin z}}\,dz = \boxed{0}$

$u=4+3\sin z$
$du=3\cos z\,dz$
$\frac{1}{3}\int_4^4 u^{-1/2}\,du$

11. $\int_0^{\pi/6} \cos^{-3} 2x \sin 2x\,dx = \boxed{\frac{3}{4}}$

$u=\cos 2x$
$du=-2\sin 2x\,dx$
$-\frac{1}{2}\int_1^{1/2} u^{-3}\,du$
$\frac{1}{2}\int_{1/2}^1 u^{-3}\,du$
$-\frac{1}{4}u^{-2}\Big]_{1/2}^1$
$-\frac{1}{4}\left(1-\left(\frac{1}{2}\right)^{-2}\right)$
$-\frac{1}{4}(1-4)$

12. $\int_0^{\pi/6} (1-\cos 3t)\sin 3t\,dt = \boxed{\frac{1}{6}}$

$u=1-\cos 3t$
$du=3\sin 3t\,dt$
$\frac{1}{3}\int_0^1 u\,du$
$\frac{1}{6}u^2\Big]_0^1 = \frac{1}{6}(1-0)$