

Calculus I

Section 5.7 – First Fundamental Theorem of Calculus

Evaluate the following definite integrals using the First Fundamental Theorem.

1. $\int_5^7 x^5 dx$

2. $\int_{-2}^3 (x+1)(x^2-1) dx$

3. $\int_8^9 \frac{3}{x^4} dx$

4. $\int_1^8 \left(\frac{2}{x^2} - x^{-4/3} \right) dx$

5. $\int_{-5}^5 \sqrt[3]{x} dx$

6. $\int_1^4 x\sqrt{x} dx$

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$$7. \int_4^9 \left(\frac{-1}{\sqrt{x}} + 2\sqrt{x} \right) dx$$

$$8. \int_0^{3\pi/2} \sin x dx$$

$$9. \int_{-\pi/4}^{\pi/4} \sec^2 x dx$$

$$10. \int_{-4}^2 |x+1| dx$$

$$11. \int_0^5 f(x) dx$$

$$f(x) = \begin{cases} -x+3, & x < 3 \\ x-3, & x \geq 3 \end{cases}$$

$$12. \int_{-3}^3 f(x) dx$$

$$f(x) = \begin{cases} x^2, & x \leq 0 \\ x^3, & x > 0 \end{cases}$$

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Evaluate the following definite integrals using the First Fundamental Theorem.

1. $\int_5^7 x^5 dx$

$$\frac{1}{6} x^6 \Big|_5^7 = \frac{1}{6} (7^6 - 5^6)$$

$$= \boxed{17004}$$

2. $\int_{-2}^3 (x+1)(x^2-1) dx$

$$\int (x^3 + x^2 - x - 1) dx$$

$$\left[\frac{1}{4} x^4 + \frac{1}{3} x^3 - \frac{1}{2} x^2 - x \right]_{-2}^3$$

$$\frac{1}{4}(3)^4 + \frac{1}{3}(3)^3 - \frac{1}{2}(3)^2 - 3 - \left[\frac{1}{4}(-2)^4 - \frac{1}{3}(-2)^3 + \frac{1}{2}(-2)^2 + (-2) \right]$$

$$= \boxed{245/12}$$

3. $\int_8^9 \frac{3}{x^4} dx = \int 3x^{-4}$

$$-x^{-3} \Big|_8^9 = \frac{-1}{9^3} + \frac{1}{8^3} = \frac{217}{373248}$$

$$\approx .00058138$$

4. $\int_1^8 \left(\frac{2}{x^2} - x^{-4/3} \right) dx$

$$\int 2x^{-2} - x^{-4/3}$$

$$-2x^{-1} + 3x^{-1/3} \Big|_1^8$$

$$-2\left(\frac{1}{8}\right) + 3 \cdot \frac{1}{8^{1/3}} + 2 - 3$$

$$= \boxed{1/4}$$

5. $\int_{-5}^5 \sqrt[3]{x} dx$

$$\int x^{1/3} dx$$

$$\frac{3}{4} x^{4/3} \Big|_{-5}^5$$

$$= \boxed{0}$$

6. $\int_1^4 x\sqrt{x} dx = \int x^{3/2}$

$$\frac{2}{5} x^{5/2} \Big|_1^4$$

$$\frac{2}{5} (4^{5/2} - 1) = \boxed{62/5}$$

7. $\int_4^9 \left(\frac{-1}{\sqrt{x}} + 2\sqrt{x} \right) dx$

$$\int -x^{-1/2} + 2x^{1/2} dx$$

$$-2x^{1/2} + \frac{4}{3} x^{3/2} \Big|_4^9$$

$$-2(3) + \frac{4}{3}(27) + 2(2) - \frac{4}{3}(8)$$

$$= \boxed{70/3}$$

8. $\int_0^{3\pi/2} \sin x dx$

$$-\cos x \Big|_0^{3\pi/2}$$

$$-\cos^{3\pi/2} + \cos 0$$

$$= \boxed{1}$$

9. $\int_{-\pi/4}^{\pi/4} \sec^2 x dx$

$$\tan x \Big|_{-\pi/4}^{\pi/4}$$

$$\tan \pi/4 - \tan(-\pi/4)$$

$$1 - (-1)$$

$$= \boxed{2}$$

10. $\int_{-4}^2 |x+1| dx$

$$\int_{-4}^{-1} -x-1 dx + \int_{-1}^2 x+1 dx$$

$$\left[-\frac{1}{2}x^2 - x \right]_{-4}^{-1} + \left[\frac{1}{2}x^2 + x \right]_{-1}^2$$

$$-\frac{1}{2}(1) + 1 + \frac{1}{2}(16) - 4 + \frac{1}{2}(4) + 2 - \frac{1}{2} + 1$$

$$= \boxed{9}$$

11. $\int_0^5 f(x) dx$

$$f(x) = \begin{cases} -x+3, & x < 3 \\ x-3, & x \geq 3 \end{cases}$$

$$\int_0^3 -x+3 dx + \int_3^5 x-3 dx$$

$$\left[-\frac{1}{2}x^2 + 3x \right]_0^3 + \left[\frac{1}{2}x^2 - 3x \right]_3^5$$

$$-\frac{1}{2}(9) + 9 + \frac{1}{2}(25) - 3(5) - \frac{1}{2}(9) + 9$$

$$= \boxed{13/2}$$

12. $\int_{-3}^3 f(x) dx$

$$f(x) = \begin{cases} x^2, & x \leq 0 \\ x^3, & x > 0 \end{cases}$$

$$\int_{-3}^0 x^2 dx + \int_0^3 x^3 dx$$

$$\left[\frac{1}{3}x^3 \right]_{-3}^0 + \left[\frac{1}{4}x^4 \right]_0^3$$

$$-\frac{1}{3}(-3)^3 + \frac{1}{4}(3)^4$$

$$= \boxed{117/4}$$