Calculus I
Section 5.3B – Integration by Substitution

Evaluate the following using *u*-substitution.

1. $\int (2x+1)^3 \, dx$

2. $\int 8\theta \sqrt[3]{\theta^2 - 1} \, d\theta$

3. $\int \sin(8z - 5) dz$

4. $\int \tan^7\left(\dfrac{x}{2}\right) \sec^2\left(\dfrac{x}{2}\right) dx$

5. $\int r^4 \left(7 - \dfrac{r^5}{10}\right)^3 dr$

6. $\int \dfrac{4y \, dy}{\sqrt{2y^2 + 1}}$

7. $\int \sec^2\left(\dfrac{x}{4}\right) dx$

8. $\int \sqrt{3-2s}\, ds$

9. $\int \theta \sqrt[4]{1-\theta^2}\, d\theta$

10. $\int 2x \sqrt[3]{x-1}\, dx$

11. $\int x^2 \sqrt{1-x}\, dx$

12. $\int \dfrac{\sec x \tan x}{\sqrt{\sec x}}\, dx$

Calculus I
Section 5.3B – Integration by Substitution

Evaluate the following using *u*-substitution.

1. $\int (2x+1)^3 \, dx$ $u = 2x+1$
$du = 2dx$

$\frac{1}{2}\int u^3 \, du$

$\frac{1}{8} u^4 + C$

$\boxed{\frac{1}{8}(2x+1)^4 + C}$

2. $\int 8\theta \sqrt[3]{\theta^2 - 1} \, d\theta$ $u = \theta^2 - 1$
$du = 2\theta \, d\theta$

$4\int u^{1/3} \, du$

$3u^{4/3} + C$

$\boxed{3(\theta^2-1)^{4/3} + C}$

3. $\int \sin(8z - 5) \, dz$ $u = 8z - 5$
$du = 8dz$

$\frac{1}{8}\int \sin u \, du$

$-\frac{1}{8}\cos u + C$

$\boxed{-\frac{1}{8}\cos(8z-5) + C}$

4. $\int \tan^7\left(\frac{x}{2}\right) \sec^2\left(\frac{x}{2}\right) dx$ $u = \tan\frac{x}{2}$
$du = \frac{1}{2}\sec^2\frac{x}{2} \, dx$

$2\int u^7 \, du$

$\frac{1}{4}u^8 + C$

$\boxed{\frac{1}{4}\tan^8\frac{x}{2} + C}$

5. $\int r^4 \left(7 - \frac{r^5}{10}\right)^3 dr$ $u = 7 - \frac{r^5}{10}$
$du = -\frac{1}{2}r^4 \, dr$

$-2\int u^3 \, du$

$-\frac{1}{2}u^4 + C$

$\boxed{-\frac{1}{2}\left(7 - \frac{r^5}{10}\right)^4 + C}$

6. $\int \frac{4y \, dy}{\sqrt{2y^2 + 1}}$ $u = 2y^2 + 1$
$du = 4y \, dy$

$\int u^{-1/2} \, du$

$2u^{1/2} + C$

$\boxed{2\sqrt{2y^2 + 1} + C}$

Calculus I
Section 5.3B – Integration by Substitution

7. $\int \sec^2\left(\dfrac{x}{4}\right) dx$ $u = \frac{1}{4}x$
   $du = \frac{1}{4} dx$

$$4\int \sec^2 u \, du$$

$$4\tan u + C$$

$$\boxed{4\tan \frac{x}{4} + C}$$

8. $\int \sqrt{3-2s}\, ds$ $u = 3-2s$
   $du = -2\, ds$

$$-\frac{1}{2}\int u^{1/2} du$$

$$-\frac{1}{3}u^{3/2}$$

$$\boxed{-\frac{1}{3}(3-2s)^{3/2} + C}$$

9. $\int \theta \sqrt[4]{1-\theta^2}\, d\theta$ $u = 1-\theta^2$
   $du = -2\theta \, d\theta$

$$-\frac{1}{2}\int u^{1/4} du$$

$$-\frac{2}{5}u^{5/4} + C$$

$$\boxed{-\frac{2}{5}(1-\theta^2)^{5/4} + C}$$

10. $\int 2x\sqrt[3]{x-1}\, dx$ $u = x-1$
   $du = dx$

$$\int 2(u+1) u^{1/3} du$$

$$2\int u^{4/3} + u^{1/3} du$$

$$\frac{6}{7}u^{7/3} + \frac{3}{2}u^{4/3} + C$$

$$\boxed{\frac{6}{7}(x-1)^{7/3} + \frac{3}{2}(x-1)^{4/3} + C}$$

11. $\int x^2\sqrt{1-x}\, dx$ $u = 1-x$
   $du = -dx$

$$-\int (1-u)^2 u^{1/2} du$$

$$-\int (1-2u+u^2) u^{1/2} du$$

$$-\int u^{1/2} - 2u^{3/2} + u^{5/2} du$$

$$-\frac{2}{3}u^{3/2} + \frac{4}{5}u^{5/2} - \frac{2}{7}u^{7/2} + C$$

$$-\frac{2}{3}(1-x)^{3/2} + \frac{4}{5}(1-x)^{5/2} - \frac{2}{7}(1-x)^{7/2} + C$$

12. $\int \dfrac{\sec x \tan x}{\sqrt{\sec x}}\, dx$ $u = \sec x$
   $du = \sec x \tan x \, dx$

$$\int u^{-1/2} du$$

$$2u^{1/2} + C$$

$$\boxed{2\sqrt{\sec x} + C}$$