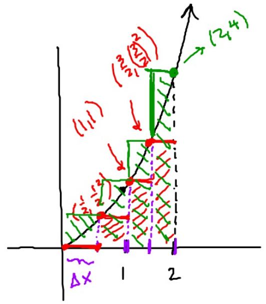
How could we find $\int_{a=0}^{b=2} x^2\,dx$? We can't! But, we can approximate it!

First, partition the x-axis.

Let's use 4 subintervals $\Rightarrow \Delta x = \frac{b-a}{n}$

n = # of subintervals



$\Delta x = \frac{2-0}{4} = \frac{1}{2}$

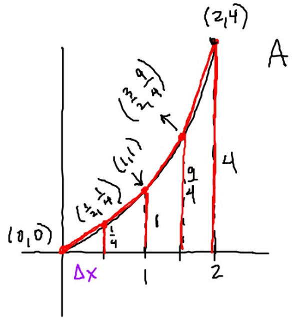
Left Rectangular Approximation Method (LRAM)

$$A = \underbrace{\tfrac{1}{2}}_{\Delta x}\left(0 + \left(\tfrac{1}{2}\right)^2 + 1 + \left(\tfrac{3}{2}\right)^2\right) = \frac{7}{4}$$

Right Rect. Approx. Method (RRAM)

$$A \approx \tfrac{1}{2}\left(\left(\tfrac{1}{2}\right)^2 + 1 + \left(\tfrac{3}{2}\right)^2 + 4\right) = 15/4$$

Trapezoidal Approx. Method (TRAP)



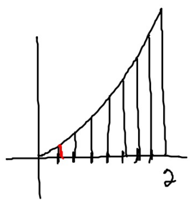
$$A = \tfrac{1}{2}\left(\tfrac{1}{2}\right)\left(0+\tfrac{1}{4}\right) + \tfrac{1}{2}\left(\tfrac{1}{2}\right)\left(\tfrac{1}{4}+1\right) + \tfrac{1}{2}\left(\tfrac{1}{2}\right)\left(1+\tfrac{9}{4}\right) + \tfrac{1}{2}\left(\tfrac{1}{2}\right)\left(\tfrac{9}{4}+4\right)$$

$$A = \tfrac{1}{2}\cdot\tfrac{1}{2}\left\{\left(0+1/4\right)+\left(\tfrac{1}{4}+1\right)+\left(1+\tfrac{9}{4}\right)+\left(\tfrac{9}{4}+4\right)\right)$$

$$= \tfrac{1}{2}\cdot\tfrac{1}{2}\left(0 + 2(1/4) + 2(1) + 2\left(\tfrac{9}{4}\right) + 4\right)$$

$$= 2.75$$

$\int_0^2 x^2\,dx \quad n=8 \quad \Rightarrow \Delta x = \frac{2-0}{8} = 1/4$



$$\text{LRAM} = \tfrac{1}{4}(0 + .0625 + .25 + .5625 + 1 + 1.5625 + 2.25 + 3.0625)$$

$$= 2.1875$$

$$\text{RRAM} = \tfrac{1}{4}(.0625 + .25 + \cdots + 3.0625 + 4)$$

$$= 3.1875$$

$$\text{TRAP} = \tfrac{1}{2}\cdot\tfrac{1}{4}\left(0 + 2(.0625) + 2(.25) + \cdots + 2(3.0625) + 4\right)$$

$$= 2.6875$$

Ex: $\int_1^5 \sin x\,dx$, $n=4 \Rightarrow \Delta x = \frac{5-1}{4} = 1$

$$\text{LRAM} = 1\,(.84147 + .9093 + .14112 + -.7568)$$

$$= 1.135$$

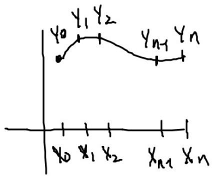
$$\text{RRAM} = 1\,(.9093 + .14112 + -.7568 + -.9589)$$

$$= -.66528$$

$$\text{TRAP} = \tfrac{1}{2}(1)\left(.84147 + 2(.9093) + 2(.14112) + 2(-.7568) + -.9589\right)$$

$$\Rightarrow .23486$$

$\int_a^b f(x)\,dx \Rightarrow \Delta x = \frac{b-a}{n}$ = HEIGHT OF RECTANGLE / TRAP. (n = # SUBINTERVALS)



$$\text{LRAM} = \Delta x\left(\overbrace{y_0 + y_1 + \cdots y_{n-1}}^{n \text{ OF THESE}}\right)$$

$$\text{RRAM} = \Delta x\left(\overbrace{y_1 + y_2 + \cdots y_n}\right)$$

$$\text{TRAP} \approx \frac{1}{2}\Delta x\left(\underbrace{y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n}_{n+1 \text{ OF THESE}}\right)$$