

Chapter 5

Section 5.3 – U Substitution

Learning Objectives

1. Given a function, recognize when basic integration techniques are not sufficient.
2. Given a function, be able to find the antiderivative using **u-substitution** – initially when u is picked for you.

Examples:

$$\int 3x^2 (x^3 + 3)^{10} dx$$

$u = x^3 + 3$
 $\frac{du}{dx} = 3x^2 dx$
 $\frac{du}{3x^2} = dx$

~~$\int 3x^2 u^{10} \frac{du}{3x^2}$~~
 $\int u^{10} du$
 $\frac{1}{11} u^{11} + C = \frac{1}{11} (x^3 + 3)^{11} + C$

Check:
 $\frac{d}{dx} \left(\frac{1}{11} (x^3 + 3)^{11} \right)$
 $= (x^3 + 3)^{10} (3x^2)$

Examples:

$$\int 4y \sqrt{1 + y^2} dy$$

$u = 1 + y^2$
 $du = 2y dy$
 $\frac{du}{2y} = dy$

~~$2 \int 4y u^{1/2} \frac{du}{2y}$~~
 $2 \int u^{1/2} du$
 $2 \cdot \frac{2}{3} u^{3/2} + C = \frac{4}{3} (1 + y^2)^{3/2} + C$

Examples:

$$\int \sec^2(2\theta) d\theta$$

$$\int \sec^2(u) \frac{du}{2}$$

$$\frac{1}{2} \int \sec^2(u) du$$

$$\frac{1}{2} \tan u + C = \frac{1}{2} \tan(2\theta) + C$$

$$u = 2\theta$$

$$du = 2 d\theta$$

$$\frac{du}{2} = d\theta$$



Examples:

$$\int (\sin^4 t) \cos t dt$$

$$\int u^4 \frac{du}{\cos t}$$

$$\int u^4 du$$

$$\frac{1}{5} u^5 + C = \frac{1}{5} \sin^5 t + C$$

$$u = \sin t$$

$$du = \cos t dt$$

$$\frac{du}{\cos t} = dt$$



Examples:

$$\int \frac{dz}{\sqrt[3]{z^2+1}}$$

$$\int \frac{du}{(u)^{4/3} \cdot 2z}$$

$$\frac{1}{2} \int u^{-1/3} du$$

$$\frac{1}{2} \cdot \frac{3}{2} u^{2/3} + C = \frac{3}{4} (z^2+1)^{2/3} + C$$

$$u = z^2 + 1$$

$$du = 2z dz$$

$$\frac{du}{2z} = dz$$



Examples:

DOUBLE SUBSTITUTION

$$\int x^2 \sqrt{x-1} dx$$

$$u = x-1 \Rightarrow u+1 = x$$

$$du = dx$$

$$\int x^2 (u)^{1/2} du$$

$$\int (u+1)^2 u^{1/2} du$$

$$\int (u^2 + 2u + 1) u^{1/2} du$$

$$\int u^{5/2} + 2u^{3/2} + u^{1/2} du$$

$$\frac{2}{7} u^{7/2} + \frac{4}{5} u^{5/2} + \frac{2}{3} u^{3/2} + C$$

$$\frac{2}{7} (x-1)^{7/2} + \frac{4}{5} (x-1)^{5/2} + \frac{2}{3} (x-1)^{3/2} + C$$



U Substitution Steps

1. Make a choice for u (how?)
 - a. Normally inside another function
 - b. It's derivative gives you something else in the problem
2. Find du .
3. Substitute u and du into the original problem.
4. Evaluate the integral.
5. Sub the original variable back in at the end.



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Homework/Classwork:

1. Classwork

- a. Section 5.2A WS
- b. Section 5.2B WS

2. Homework

- a. Day 1: p. 261 #1-3
- b. Day 2: p. 261 #5-27 odd, 31, 33

