Chapter 5

Section 5.2 - Antiderivatives

"You must unlearn what you have learned."

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Antiderivative

A function F(x) is an ***antiderivative*** of f(x) if

$$F'(x) = f(x)$$

Examples: Find F(x) given the functions f(x) below:

$f(x) = x^2 \Rightarrow F(x) = \frac{1}{3}x^3 + C$

$f(x) = 3x^5 \Rightarrow F(x) = \frac{1}{2}x^6 + C$

EACH ANSWER COULD HAVE A CONSTANT ON THE END.

$f(x) = \cos x \Rightarrow F(x) = \sin x + C$

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Learning Objectives

1. Given a function, be able to find the antiderivative.
2. Certain antiderivatives must be memorized – make sure you do so (I smell a pop quiz.)
3. Given a function, rewrite into a form such that finding an antiderivative is possible.
4. Make sure you understand the new notation associated with **integration**.

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Integration:

- The process of finding antiderivatives
- New notation – tells us when we need to find an antiderivative

$$\underbrace{\int}_{\text{INTEGRATION SYMBOL}} \overbrace{x^2}^{\text{INTEGRAND}} \underbrace{dx}_{\text{"WITH RESPECT TO } x\text{"}} = \overbrace{\frac{1}{3}x^3}^{\text{AN ANTIDERIVATIVE OF INTEGRAND}} + \underbrace{C}_{\text{CONSTANT OF INTEGRATION}}$$

Differentiation	Integration
$\frac{d}{dx}(x) = 1$	$\int 1\,dx = x + C$
$\frac{d}{dx}\left(\frac{1}{n+1}x^{n+1}\right) = x^n$	$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$ POWER RULE $(n \neq -1)$
$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x\,dx = -\cos x + C$

Differentiation	Integration
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\int \csc^2 x\,dx = -\cot x + C$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\int \csc x \cot x\,dx = -\csc x + C$

Examples:

$$\int \left(2x^7 - 3\sec^2 x\right) dx$$

$$2 \cdot \frac{1}{8}x^8 - 3\tan x + C$$

$$\boxed{\frac{1}{4}x^8 - 3\tan x + C}$$

$$\int \left(\frac{1}{x^4} + \sqrt[3]{x}\right) dx$$

$$\int x^{-4} + x^{1/3}\,dx$$

$$\boxed{-\frac{1}{3}x^{-3} + \frac{3}{4}x^{4/3} + C}$$

Properties of Integrals

$$\frac{d}{dx}\left[\int f(x)dx\right]= f(x)$$

$$\int(cf(x))dx = c\int f(x)\,dx$$

$$\int(f(x)\pm g(x))dx = \int f(x)\,dx \pm \int g(x)\,dx$$

Examples:

$$\int(x+2)(x+3)dx$$

$$\int x^2+5x+6\;dx$$

$$\boxed{\frac{1}{3}x^3+\frac{5}{2}x^2+6x+C}$$

$$\int\left(\frac{x+1}{x^3}\right)dx$$

$$\int \frac{x}{x^3}+\frac{1}{x^3}\,dx$$

$$\int x^{-2}+x^{-3}\,dx$$

$$\boxed{-x^{-1}-\frac{1}{2}x^{-2}+C}$$

Examples:

$\sin^2 x+\cos^2 x=1$
$1+\cot^2 x=\csc^2 x$
$1+\tan^2 x=\sec^2 x$

$$\int\sqrt{x}(x+3)dx$$

$$\int x^{1/2}(x+3)\,dx$$

$$\int x^{3/2}+3x^{1/2}\,dx$$

$$\boxed{\frac{2}{5}x^{5/2}+2x^{3/2}+C}$$

$$\int\cot^2 x\,dx$$

REWRITE W/ IDENTITY

$$\int \csc^2 x-1\,dx$$

$$\boxed{-\cot x-x+C}$$

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Homework/Classwork:

1. **Classwork – Section5.2 WS**
2. **Homework – p. 256 #1-29 odd, 45, 47**