

$$1) x^3 + y^3 = 18xy$$

$$3x^2 + 3y^2 \frac{dy}{dx} = 18x \frac{dy}{dx} + y18$$

$$3y^2 \frac{dy}{dx} - 18x \frac{dy}{dx} = 18y - 3x^2$$

$$(3y^2 - 18x) \frac{dy}{dx} = 18y - 3x^2$$

$$\boxed{\frac{dy}{dx} = \frac{18y - 3x^2}{3y^2 - 18x}}$$

$$2) x^2y + xy^2 = 6$$

$$x^2 \left[ \frac{dy}{dx} \right] + y[2x] + x2y \frac{dy}{dx} + y^2 = 0$$

$$x^2 \frac{dy}{dx} + 2xy \frac{dy}{dx} + 2xy + y^2 = 0$$

$$x^2 \frac{dy}{dx} + 2xy \frac{dy}{dx} = -2xy - y^2$$

$$\frac{dy}{dx} (x^2 + 2xy) = -2xy - y^2$$

$$\boxed{\frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy}}$$

$$3) x^3 - xy + y^3 = 1$$

$$3x^2 - x \left[ \frac{dy}{dx} \right] + y[1] + 3y^2 \left[ \frac{dy}{dx} \right] = 0$$

$$3x^2 - x \frac{dy}{dx} + y + 3y^2 \frac{dy}{dx} = 0$$

$$3y^2 \frac{dy}{dx} - x \frac{dy}{dx} = -3x^2 - y$$

$$(3y^2 - x) \frac{dy}{dx} = -3x^2 - y$$

$$\boxed{\frac{dy}{dx} = \frac{-3x^2 - y}{3y^2 - x}}$$

$$4) x + y = y^2 + 2xy$$

$$1 + \frac{dy}{dx} = 2y \frac{dy}{dx} + 2x \left[ \frac{dy}{dx} \right] + y[2]$$

$$1 + \frac{dy}{dx} = 2y \frac{dy}{dx} + 2x \frac{dy}{dx} + 2y$$

$$-\frac{dy}{dx} + 2y \frac{dy}{dx} + 2x \frac{dy}{dx} = 1 - 2y$$

$$(-1 + 2y + 2x) \frac{dy}{dx} = 1 - 2y$$

$$\boxed{\frac{dy}{dx} = \frac{1 - 2y}{2x + 2y - 1}}$$

$$5) (3xy + 7)^2 = 6xy$$

$$2(3xy + 7) \left[ 3x \frac{dy}{dx} + y[3] \right] = 6 \frac{dy}{dx}$$

$$2(3xy + 7)(3x \frac{dy}{dx} + 3y) = 6 \frac{dy}{dx}$$

$$(6xy + 14)(3x \frac{dy}{dx} + 3y) = 6 \frac{dy}{dx}$$

$$18x^2y \frac{dy}{dx} + 18xy^2 + 42x \frac{dy}{dx} + 42y = 6 \frac{dy}{dx}$$

$$18x^2y \frac{dy}{dx} + 42x \frac{dy}{dx} - 6 \frac{dy}{dx} = -18xy^2 - 42y$$

$$6 \frac{dy}{dx} (3x^2y + 7x - 1) = 6(-3xy^2 - 7y)$$

$$\boxed{\frac{dy}{dx} = \frac{-3xy^2 - 7y}{3x^2y + 7x - 1}}$$

$$6) 4 - x^3y^2 = 3xy$$

$$-x^3 \left[ 2y \frac{dy}{dx} \right] + y^2 \left[ 3x^2 \right] = 3 \frac{dy}{dx}$$

$$-2x^3y \frac{dy}{dx} + 3x^2y^2 = 3 \frac{dy}{dx}$$

$$3x^2y^2 = 3 \frac{dy}{dx} + 2x^3y \frac{dy}{dx}$$

$$3x^2y^2 = (3 + 2x^3y) \frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{3x^2y^2}{3 + 2x^3y}}$$

$$7) 4x - \cos y = 2y^2$$

$$4 + \sin y \frac{dy}{dx} = 4y \frac{dy}{dx}$$

$$\sin y \frac{dy}{dx} - 4y \frac{dy}{dx} = -4$$

$$(\sin y - 4y) \frac{dy}{dx} = -4$$

$$\boxed{\frac{dy}{dx} = \frac{-4}{\sin y - 4y}}$$

$$8) \cot(xy) + 4 = 2x$$

$$-\csc^2(xy) \left[ x \frac{dy}{dx} + y \right] = 2$$

$$-\csc^2 xy \cdot x \frac{dy}{dx} - y \csc^2 xy = 2$$

$$-x \csc^2 xy \frac{dy}{dx} = 2 + y \csc^2 xy$$

$$\boxed{\frac{dy}{dx} = \frac{2 + y \csc^2 xy}{-x \csc^2 xy}}$$

$$10) x = \tan(2y)$$

$$1 = \sec^2(2y) \cdot 2 \frac{dy}{dx}$$

$$\frac{dy}{dx} (2 \sec^2 2y) = 1$$

$$\boxed{\frac{dy}{dx} = \frac{1}{2 \sec^2(2y)}}$$

$$11) x = 5 \sin y$$

$$1 = \cos y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

$$\boxed{\frac{dy}{dx} = \sec y}$$

$$12) 4x \sin y + y^2 \cos x = 3$$

$$4x \cos y \frac{dy}{dx} + 4 \sin y + y^2 (-\sin x) + \cos x \cdot 2y \frac{dy}{dx} = 0$$

$$4x \cos y \frac{dy}{dx} + 2y \cos x \frac{dy}{dx} = y^2 \sin x - 4 \sin y$$

$$(4x \cos y + 2y \cos x) \frac{dy}{dx} = y^2 \sin x - 4 \sin y$$

$$\boxed{\frac{dy}{dx} = \frac{y^2 \sin x - 4 \sin y}{4x \cos y + 2y \cos x}}$$

$$13) 2y = \frac{x-1}{x+1}$$

$$2 \frac{dy}{dx} = \frac{(x+1)[1] - (x-1)[1]}{(x+1)^2}$$

$$2 \frac{dy}{dx} = \frac{x+1 - x+1}{(x+1)^2}$$

$$2 \frac{dy}{dx} = \frac{2}{(x+1)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{(x+1)^2}}$$

$$14) 2xy + \pi \sin y = 2\pi$$

$$2x \frac{dy}{dx} + 2y + \pi \cos y \frac{dy}{dx} = 0$$

$$2x \frac{dy}{dx} + \pi \cos y \frac{dy}{dx} = -2y$$

$$(2x + \pi \cos y) \frac{dy}{dx} = -2y$$

$$\boxed{\frac{dy}{dx} = \frac{-2y}{2x + \pi \cos y}}$$

$$15) x \sin 2y = y \cos 3x$$

$$x [2 \cos 2y \frac{dy}{dx}] + \sin 2y = y (-3 \sin x) + \cos 3x \frac{dy}{dx}$$

$$2x \cos 2y \frac{dy}{dx} + \sin 2y = -3y \sin x + \cos 3x \frac{dy}{dx}$$

$$2x \cos 2y \frac{dy}{dx} - \cos 3x \frac{dy}{dx} = -3y \sin x - \sin 2y$$

$$(2x \cos 2y - \cos 3x) \frac{dy}{dx} = -3y \sin x - \sin 2y$$

$$\boxed{\frac{dy}{dx} = \frac{-3y \sin x - \sin 2y}{2x \cos 2y - \cos 3x}}$$

$$16) x^2 \cos^2 y - \sin y = 0$$

$$x^2 [2 \cos y] [-\sin y \frac{dy}{dx}] + \cos^2 y \cdot 2x$$

$$- \cos y = 0$$

$$(-2x^2 \sin y \cos y) \frac{dy}{dx} = \cos y - 2x \cos^2 y$$

$$\boxed{\frac{dy}{dx} = \frac{\cos y - 2x \cos^2 y}{-2x^2 \sin y \cos y}}$$

$$17) \tan x - 2xy = y^3$$

$$\sec^2 x - 2x \frac{dy}{dx} + y(-2) = 3y^2 \frac{dy}{dx}$$

$$\sec^2 x - 2x \frac{dy}{dx} - 2y = 3y^2 \frac{dy}{dx}$$

$$3y^2 \frac{dy}{dx} + 2x \frac{dy}{dx} = \sec^2 x - 2y$$

$$(3y^2 + 2x) \frac{dy}{dx} = \sec^2 x - 2y$$

$$\boxed{\frac{dy}{dx} = \frac{\sec^2 x - 2y}{3y^2 + 2x}}$$

$$23) 6x^2 + 3xy + 2y^2 + 17y = 6 \quad (-1, 0)$$

$$12x + 3x \frac{dy}{dx} + 3y + 4y \frac{dy}{dx} + 17 \frac{dy}{dx} = 0$$

$$3x \frac{dy}{dx} + 4y \frac{dy}{dx} + 17 \frac{dy}{dx} = -12x - 3y$$

$$(3x + 4y + 17) \frac{dy}{dx} = -12x - 3y$$

$$\frac{dy}{dx} = \frac{-12x - 3y}{3x + 4y + 17}$$

$$\frac{dy}{dx} = \frac{12}{-3+17} = \frac{12}{14} = \frac{6}{7} \quad y-0 = \frac{6}{7}(x+1)$$

$$\boxed{y = \frac{6}{7}x + \frac{6}{7}}$$

$$18) x^2 + y^2 = 4$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{y(-1) - (-x)(\frac{dy}{dx})}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{-y + x \frac{dy}{dx}}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{-y + x(-\frac{x}{y})}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{-y - \frac{x^2}{y}}{y^2}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{-y^2 - x^2}{y^3}}$$

$$19) y^2 = 2x + x^2$$

$$2y \frac{dy}{dx} = 2 + 2x$$

$$\frac{dy}{dx} = \frac{1+x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{y[1] - (1+x)\frac{dy}{dx}}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{y}{y} - (1+x)(\frac{1+x}{y})}{y^2}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{y^2 - x^2 - 2x - 1}{y^3}}$$

$$20) y^2 + 2y = 2x + 1$$

$$2y \frac{dy}{dx} + 2 \frac{dy}{dx} = 2$$

$$y \frac{dy}{dx} + \frac{dy}{dx} = 1$$

$$(y+1) \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{y+1}$$

$$\frac{d^2y}{dx^2} = \frac{(y+1)0 - 1(\frac{dy}{dx})}{(y+1)^2}$$

$$\frac{d^2y}{dx^2} = \frac{-\frac{dy}{dx}}{(y+1)^2}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{-1}{(y+1)^3}}$$

$$21) x^2 + xy - y^2 = 1 \quad (2, 3)$$

$$2x + x \frac{dy}{dx} + y - 2y \frac{dy}{dx} = 0$$

$$x \frac{dy}{dx} - 2y \frac{dy}{dx} = -2x - y$$

$$(x-2y) \frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x - y}{x - 2y}$$

$$\frac{dy}{dx} = \frac{-2(2) - 3}{-2 - 2(3)} = \frac{-7}{-8} = \frac{7}{8}$$

$$y - 3 = \frac{7}{8}(x - 2)$$

$$y - 3 = \frac{7}{8}x - \frac{7}{4}$$

$$\boxed{y = \frac{7}{8}x + \frac{5}{4}}$$

$$22) x^2 y^2 = 9 \quad (3, -4)$$

$$x^2 [2y \frac{dy}{dx}] + y^2 [2x] = 0$$

$$2x^2 y \frac{dy}{dx} + 2xy^2 = 0$$

$$2x^2 y \frac{dy}{dx} = -2xy^2$$

$$\frac{dy}{dx} = \frac{-2xy^2}{2x^2y}$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

$$\rightarrow \frac{y}{x}$$

$$y + 4 = \frac{4}{3}(x - 3)$$

$$y + 4 = \frac{4}{3}x - 4$$

$$\boxed{y = \frac{4}{3}x - 8}$$