① $y = -2\sin^2(x^3) = -2[\sin(x^3)]^2$

$y' = -4(\sin x^3)[\cos(x^3)][3x^2]$

$y' = -12x^3(\sin x^3)(\cos x^3)$

② $y = \tan^2(x^3+2) = [\tan(x^3+2)]^2$

$y' = 2(\tan(x^3+2))[\sec^2(x^3+2)][3x^2]$

$y' = 6x^2(\tan(x^3+2))(\sec^2(x^3+2))$

③ $y = (\sin x + \cos x)(\sin x - \cos x)$

$y = \sin^2 x - \cos^2 x$

$y = (\sin x)^2 - (\cos x)^2$

$y' = 2\sin x[\cos x] - 2\cos x[-\sin x]$

$y' = 4\sin x \cos x$

④ $y = \frac{2x}{\sin x} + \frac{\sin x}{3x}$

$y' = \frac{\sin x[2] - 2x[\cos x]}{\sin^2 x} + \frac{3x[\cos x] - \sin x[3]}{(3x)^2}$

$y' = \frac{2\sin x - 2x\cos x}{\sin^2 x} + \frac{3x\cos x - 3\sin x}{9x^2}$

$y' = \frac{2\sin x - 2x\cos x}{\sin^2 x} + \frac{x\cos x - \sin x}{3x^2}$

⑤ $y = \csc(2x) \cdot \sec(4x)$

$y' = \csc(2x)[\sec 4x \tan 4x][4] + \sec(4x)[-\csc 2x \cot 2x][2]$

$y' = 4\csc(2x)\sec(4x)\tan(4x) - 2\sec(4x)\csc(2x)\cot(2x)$

⑥ $y = 2\sqrt{x} + 4\tan^2(x^2)$

$y = 2x^{1/2} + 4(\tan x^2)^2$

$y' = x^{-1/2} + 8(\tan x^2)[\sec^2 x^2][2x]$

$y' = \frac{1}{\sqrt{x}} + 16x \tan x^2 \sec^2 x^2$

⑦ $y = -\cot^3(4x)$

$y = -(\cot(4x))^3$

$y' = -3(\cot(4x))^2[-\csc^2(4x)][4]$

$y' = 12\cot^2 4x \csc^2 4x$

(8) $y = 3x(\sec x + 1)$

$y' = 3x[\sec x \tan x] + (\sec x + 1)[3]$

$y' = 3x\sec x \tan x + 3(\sec x + 1)$

(9) $y = \dfrac{2\tan x - 5}{\sec x}$ REWRITE ➡ $\dfrac{2\frac{\sin x}{\cos x} - 5}{\frac{1}{\cos x}}$

$y = 2\sin x - 5\cos x$ *

$y' = 2\cos x + 5\sin x$ $= 2\sin x - 5\cos x$ *

OR

$y' = \dfrac{\sec x[2\sec^2 x] - (2\tan x - 5)[\sec x \tan x]}{\sec^2 x}$

$y' = \dfrac{2\sec^3 x - 2\sec x \tan^2 x + 5\sec x \tan x}{\sec^2 x}$

$y' = \dfrac{2\sec x(\sec^2 x - \tan^2 x) + 5\sec x \tan x}{\sec^2 x}$

$y' = \dfrac{2\sec x + 5\sec x \tan x}{\sec^2 x} = \dfrac{\sec x(2 + 5\tan x)}{\sec^2 x}$

$y' = \dfrac{2 + 5\tan x}{\sec x} = \dfrac{2}{\sec x} + \dfrac{5\tan x}{\sec x} = 2\cos x + 5\sin x$ ✓

(10) $y = \sec x(\sin x + \cos x)$

$y = \tan x + 1$

$y' = \sec^2 x$

(11) $y = \cos x \cdot \sin(2x)$

$y' = \cos x[\cos(2x)[2]] + \sin(2x)[-\sin x]$

$y' = 2\cos x \cos(2x) - \sin x \sin(2x)$

(12) $y = x^3\cos x - x^3\sin x$

$y = x^3(\cos x - \sin x)$

$y' = x^3[-\sin x - \cos x] + (\cos x - \sin x)[3x^2]$

$y' = -x^3\sin x - x^3\cos x + 3x^2\cos x - 3x^2\sin x$

OR

$y = x^3\cos x - x^3\sin x$

$y' = x^3[-\sin x] + \cos x[3x^2] - [x^3[\cos x] + \sin x[3x^2]]$

$y' = -x^3\sin x + 3x^2\cos x - x^3\cos x - 3x^2\sin x$

(13) $y = x^4 - 3\cos(5x)$

$y' = 4x^3 + 3\sin 5x\,[5]$

$y' = 4x^3 + 15\sin 5x$

(14) $y = \sec^3(8x^2)$

$y' = 3\sec^2 8x^2\,[\sec 8x^2 \tan 8x^2]\,[16x]$

$y' = 48x\sec^2 8x^2 \sec 8x^2 \tan 8x^2$

$y' = 48x\sec^3 8x^2 \tan 8x^2$

(15) $y = \sin(\cos(2x-3))$

$y' = \cos(\cos(2x-3))\,[-\sin(2x-3)]\,[2]$

$y' = -2\cos(\cos(2x-3))(\sin(2x-3))$

(16) $y = 2\csc x + 4x^2 - 3$

$y = -2\csc x \cot x + 8x$

(17) $y = \dfrac{4 - 3\csc x}{\csc x} = 4\sin x - 3$

$y' = 4\cos x$

(18) $y = 4\sec\sqrt{x} = 4\sec x^{1/2}$

$y' = 4\sec\sqrt{x}\tan\sqrt{x}\,[\frac{1}{2}x^{-1/2}]$

$y' = \dfrac{2\sec\sqrt{x}\tan\sqrt{x}}{\sqrt{x}}$

(19) $y = 3x^2\csc x$

$y' = 3x^2[-\csc x\cot x] + \csc x[6x]$

$y' = -3x^2\csc x\cot x + 6x\csc x$

(20) $y = 4[\csc x^{2/3}]^5$

$y' = 20(\csc x^{2/3})^4\,[-\csc x^{2/3}\cot x^{2/3}]\,[\frac{2}{3}x^{-1/3}]$

$y' = \frac{-40}{3}(\csc x^{2/3})^4(\csc x^{2/3})(\cot x^{2/3})\,x^{-1/3}$

$y' = \dfrac{-40(\csc x^{2/3})^5(\cot x^{2/3})}{3\sqrt[3]{x}}$

(21) $y = 4x\tan x + 3x\sec x$

$y' = 4x[\sec^2 x] + \tan x[4] + 3x[\sec x\tan x] + \sec x[3]$

$y' = 4x\sec^2 x + 4\tan x + 3x\sec x\tan x + 3\sec x$

(22) $y = \dfrac{-\cos x}{\sin x} + 4\csc x$

$y = -\cot x + 4\csc x$

$y = \csc^2 x - 4\csc x\cot x$

(23) $y = -(\sin(2x^2+x-1))^3$

$y' = -3(\sin(2x^2+x-1))^2\,[\cos(2x^2+x-1)]\,[4x+1]$

$y' = (-12x-3)\cos(2x^2+x-1)\sin^2(2x^2+x-1)$

(24) $y = \cot(4x+x^3)$

$y' = -\csc^2(4x+x^3)[4+3x^2]$

$y' = -(4+3x^2)\csc^2(4x+x^3)$

(25) $y = 3\sec x + 2\tan x$

$y' = 3\sec x\tan x + 2\sec^2 x$

(26) $y = -3(\csc(-x^{-1}+x))$

$y' = 3.\csc\left(\frac{1}{x}+x\right)\cot\left(\frac{1}{x}+x\right)\left[x^{-2}+1\right]$

$y' = 3\left(\frac{1}{x^2}+1\right)\csc\left(\frac{1}{x}+x\right)\cot\left(\frac{1}{x}+x\right)$

(27) $y = \dfrac{\cot x}{1+\cot x}$

$y' = \dfrac{(1+\cot x)[-\csc^2 x] - \cot x[-\csc^2 x]}{(1+\cot x)^2}$

$y' = \dfrac{-\csc^2 x - \cot x\csc^2 x + \cot x\csc^2 x}{(1+\cot x)^2}$

$y' = \dfrac{-\csc^2 x}{(1+\cot x)^2}$

(28) $y = \cos(\sin 2x)$

$y' = -\sin(\sin 2x)[\cos(2x)][2]$

$y' = -2\sin(\sin 2x)(\cos 2x)$

(29) $y = \dfrac{\sin^2 x}{(1+\cos x)^2} = \dfrac{(\sin x)^2}{(1+\cos x)^2}$

$y' = \dfrac{(1+\cos x)[2\sin x\cos x] - \sin^2 x[2(1+\cos x)[-\sin x]]}{(1+\cos x)^4}$

$y' = \dfrac{2\sin x\cos x + 2\sin x\cos^2 x + 2\sin^3 x + 2\sin^3 x\cos x}{(1+\cos x)^4}$

(30) $y = 2\tan(x^2)\sec 3x$

$y' = 2\tan(x^2)[3\sec 3x\tan 3x] + \sec 3x[2\sec^2(x^2)][2x]$

$y' = 6\tan x^2\sec 3x\tan 3x + 4x\sec^2(x^2)\sec 3x$

(31) $y = 4\cot(3x-1)^3$

$y' = -4\csc^2(3x-1)^3\left[3(3x-1)^2[3]\right]$

$y' = -\csc^2(3x-1)^3(3x-1)^2$
$y' = -(3x-1)^2\csc^2(3x-1)^3$

(32) $y = 5\sin(x^3)(\cos x)^2$

$y' = 5\sin x^3\left[2\cos x(-\sin x)\right] + \cos^2 x\left[5\cos(x^3)[3x^2]\right]$
$y' = -10\sin x^3\sin x\cos x + 15x^2\cos^2 x\cos x^3$

(33) $y = \frac{\sin x}{\cos x} + \frac{\cos x}{\cos x}$
$y = \tan x + 1$
$y' = \sec^2 x$

(34) $y = -3\cos x + 1 \qquad x = \pi$

$y = -3\cos\pi + 1$
$y = 4$

$y' = 3\sin x$
$y'(\pi) = 3\sin\pi$
$= 0$

$\boxed{y = 4}$

(35) $y = 2\cos x - \sin x \qquad x = \frac{\pi}{2}$

$y = 2\cos\frac{\pi}{2} - \sin\frac{\pi}{2}$
$y = 0 - 1$
$y = -1$

$y' = -2\sin x - \cos x$
$y' = -2\sin\left(\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{2}\right)$
$y' = -2(1) - 0$
$y' = -2$

$y = mx + b$

$(-1) = -2\left(\frac{\pi}{2}\right) + b$
$-1 = -\pi + b$
$b = \pi - 1$

$\boxed{y = -2x + \pi - 1}$