Logarithmic Functions

Section 3.2
Definition of a Logarithm

\[ y = b^x \iff \log_b y = x \]

"IF AND ONLY IF"

EXPONENTIAL FORM

LOGARITHMIC FORM

A LOGARITHM IS AN EXPONENT
Evaluate the following – it helps to rewrite in exponential form.

\[ \log_4 16 = x \]
\[ 4^x = 16 \]
\[ x = 2 \]

\[ \log_2 64 = x \]
\[ 2^x = 64 \]
\[ x = 6 \]

\[ \log_5 1 = x \]
\[ 5^x = 1 \]
\[ x = 0 \]

\[ \log_3 \frac{1}{81} = x \]
\[ 3^x = \frac{1}{81} \]
\[ = \frac{1}{3^4} \]
\[ x = -4 \]
Evaluate the following – it helps to rewrite in exponential form.

**Comm. Logs (Base 10)**

<table>
<thead>
<tr>
<th>log 1 = x</th>
<th>log 10</th>
<th>log 1000</th>
<th>log 0.0001</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10^x = 1)</td>
<td>(10^x = 10)</td>
<td>(10^x = 1000)</td>
<td>(10^x = 0.0001)</td>
</tr>
<tr>
<td>(x = 0)</td>
<td>(x = 1)</td>
<td>(x = 3)</td>
<td>(x = -4)</td>
</tr>
</tbody>
</table>
Properties of Logarithms:

\[
\log_b 1 = 0
\]

\[
\log_b b = 1
\]

\[
\log_b b^{f(x)} = f(x)
\]

If \( \log_b x = \log_b y \), then \( x = y \)

\[
\log_4 \left( \frac{3x+5}{5} \right) = \log_4 (x)
\]

\[
3x + 5 = x
\]

\[
x = 5/2
\]

\[
\text{ONE-TO-ONE PROPERTY}
\]

\[
\text{If } \log_b x = \log_b y, \text{ then } x = y
\]
Graphs of Logarithmic Functions: \( f(x) = \log_b x \quad b > 1 \)

Domain: \((0, \infty)\)

Range: \((\infty, \infty)\)

Y-intercept: \(\emptyset\)

X-intercept(s): \((1, 0)\)

Increasing/Decreasing: \((0, \infty)\)
Natural Logarithmic Function: \( f(x) = \ln x \)

Domain: \((0, \infty)\)

Range: \((-\infty, \infty)\)

Y-intercept: \(\emptyset\)

X-intercept(s): \((1, 0)\)

Increasing/Decreasing: \((0, \infty)\)
Properties of Natural Logarithms:

\[
\ln 1 = 0
\]

\[
\ln e = 1
\]

\[
\ln e^{f(x)} = f(x)
\]

If \( \ln x = \ln y \), then \( \boxed{x = y} \)

\[
\ln x = \ln (x^2 - 2x - 4)
\]

\[
\begin{align*}
\ln x &= \ln (x^2 - 2x - 4) \\
x &= x^2 - 2x - 4 \\
0 &= x^2 - 3x - 4 \\
0 &= (x - 4)(x + 1)
\end{align*}
\]

\( x = 4 \) \( \times \) \( \text{not in domain} \) \( (0, \infty) \)
Domains involving Logarithmic Function

\[ f(x) = \ln(x + 3) \]

\[ D: x + 3 > 0 \quad \Rightarrow \quad x > -3 \]

\[ f(x) = -3\ln(x^2 - 3x + 2) \]

\[ D: x^2 - 3x + 2 > 0 \]

\[ (x-2)(x-1) \]

\[ \frac{+}{+} \quad \frac{-}{-} \quad \frac{+}{+} \]

\[ 1 \quad 2 \]

\[ D: (-\infty, 1) \cup (2, \infty) \]
Application:

Suppose the function $s(t)$ below models a student’s score on a math retest as a function of time, $t$, in months taken after the original test.

$$s(t) = 78 - \log(t + 1)$$

a. What did the student score on the original test?

$$s(0) = 78 - \log(1) = 78$$

b. What is the student’s score on a retest 3 months later?

$$s(3) = 78 - \log(4) = 77.398$$

c. What is the student’s score on a retest 1 year later?

$$s(12) = 78 - \log(13) = 76.886$$
Homework: